Pay-as-Bid versus Marginal Pricing–Part II: Market Behavior Under Strategic Generator Offers

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***Abstract*—As the arguments for and against the use of pay-as-bid (PAB) or marginal pricing (MP) in electricity pools tend to be qualitative, we compare the quantitative behavior of the two markets assuming that generators submit the best strategic offers that correspond to the specified pricing method. In Part I of this two-part study, assuming that the system marginal costs for PAB and MP are random with known probability density functions, we develop generator strategic offers by maximizing the corresponding expected values of the generator profits over the offer parameters. In Part II, relations are established between the system marginal costs for each market type and a common random demand, thus allowing the two markets to be compared through the expected values and variances of the individual generation profits and of the consumer payments.**

This comparison demonstrates both theoretically and through simulation that: 1) he expected values of the individual generator profits as well as of the consumer payments are the same under MP and PAB and 2) the variances of the individual generator profits and of the consumer payments however are larger under MP than under PAB. The primary conclusion is then that although MP and PAB yield identical expected generator profits and consumer payments, the risk of not meeting these expected values is greater under MP than under PAB.

***Index Terms*—Expected profit, marginal pricing, pay-as-bid, perfectly competitive markets, strategic offers, system marginal cost, uncertainty.**

I. Introduction

WHETHER "pay as bid" (PAB) pricing can or should replace the more common marginal pricing (MP) in electricity markets is the subject of an on-going spirited debate [1]–[5]. Since this discussion is rooted on qualitative or experimental arguments, we have conducted a two-part theoretical and quantitative study comparing the behavior of electricity markets under both pricing approaches.

In the first part of the comparative study [6], we propose strategic generator offers under the assumption that the system marginal costs are random with known probability density functions. For each pricing method we then derive strategic generator offers that maximize their individual expected profits. These strategic offers which can be found in analytic form are clearly distinct from one pricing method to the other.

This second part of this study recognizes that the relation between the system marginal cost (SMC) and the demand is affected by the pricing method and by its corresponding strategic offers. One way to derive this relationship is to assume that the system supply curve is known [10]. Here, however, the SMC versus demand relation is derived from basic principles and from the definitions of the two types of strategic offers. The next necessary step to make a fair comparison is to establish a common link between the two pricing methods, which here is done via common demand conditions taken to be a random variable lying within a known uncertainty range.

The two markets are then compared in terms of the corresponding expected values and variances of the generation profits and consumer payments, all of which can be computed analytically and verified through simulation.

This comparison demonstrates that: 1) the expected values of the individual generator profits and of the consumer payments are the same under MP and PAB and 2) the variances of the individual generator profits and of the consumer payments are however larger under MP than under PAB.

The main conclusion is then that although MP and PAB yield identical expected generator profits and consumer payments, the risk of not meeting these expected values is greater under MP than under PAB.

II. Summary of Strategic Offers

First, we present a brief summary of the main results of Part I [6].

Under PAB or MP, the power pool schedules and dispatches the generators by minimizing the total cost of the submitted offers subject to the power balance constraint and to the generator limits, that is

$$\min_{u,P_g} \sum_i C_i(P_{gi}, a_i) \tag{1}$$

where

$$\sum_i P_{gi} = P_d(\lambda) \tag{2}$$

and

$$u_i P_{gi}^{\min} \le P_{gi} \le u_i P_{gi}^{\max}. \tag{3}$$

The binary $0/1$ variable u_i determines whether the unit is scheduled on or off. To simplify the analysis, we assume that the generator cost offers are of the form[1]

$$C_i(P_{gi}, a_i) = a_i P_{gi}. \tag{4}$$

We also assume that the generator limits are always fixed at their true values, in other words that generators do not withhold power as a gaming strategy. Thus, the only parameter that

[1]As shown in [6], more general multi-segment offers with fixed and variable costs could also be submitted but the analysis is not as transparent.

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can be varied by a generator when defining its offer strategy is the incremental cost a_i. The true value of the incremental cost, known only to the generator itself, is denoted by a_i^*.

Under the perfect market assumption used here, the System Marginal Cost (SMC), in other words, the Lagrange multiplier associated with the power balance (2), λ, cannot be affected by any individual generator offer a_i. Then, the optimal generation schedule and dispatch that solves (1)–(3), also known as the market clearing solution, can be found in terms of λ and the offer a_i, [7]. The on/off schedule of unit i is

$$u_i(\lambda, a_i) = \begin{cases} 1, & \text{if } \lambda \geq a_i \\ 0, & \text{if } \lambda < a_i \end{cases} \tag{5}$$

while the power output when $\lambda > a_i$ is

$$P_{gi}(\lambda, a_i) = u_i(\lambda, a_i) P_{gi}^{\max}. \tag{6}$$

The marginal units whose offers a_i are equal to the system marginal cost, λ, operate somewhere between their upper and lower limits. However, since under the perfect market assumption the maximum output of any generator is negligible compared to the system demand, the marginal generators can also be assumed to operate at their maximum output with negligible error.

The same market clearing solution applies to both PAB and MP, however with corresponding SMCs and profit-maximizing strategies that may be different. Under PAB pricing, for an arbitrary SMC, λ^{PAB}, and offer parameter a_i^{PAB}, the profit function of unit i is given by

$$\begin{aligned} &\mathrm{pr}_i^{\mathrm{PAB}}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right) \\ &\quad = C_i\left(P_{gi}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right), a_i^{\mathrm{PAB}}\right) \\ &\quad\quad - C_i\left(P_{gi}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right), a_i^*\right) \\ &\quad = u_i\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right)\left(a_i^{\mathrm{PAB}} - a_i^*\right) P_{gi}^{\max} \end{aligned} \tag{7}$$

while under MP, the profit function is

$$\begin{aligned} \mathrm{pr}_i^{\mathrm{MP}}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right) &= \lambda^{\mathrm{MP}} P_{gi}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right) \\ &\quad - C_i\left(P_{gi}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right), a_i^*\right) \\ &= u_i\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right)\left(\lambda^{\mathrm{MP}} - a_i^*\right) P_{gi}^{\max}. \end{aligned} \tag{8}$$

This paper assumes that under both PAB and MP the predicted load is random but known to lie inside the common range, $[P_d^{\min}, P_d^{\max}]$. This demand uncertainty results in corresponding uncertainty ranges in the SMCs of both pricing methods that are generally different, that is, $\lambda_{\min}^{\mathrm{MP}} \leq \lambda^{\mathrm{MP}} \leq \lambda_{\max}^{\mathrm{MP}}$ for MP and $\lambda_{\min}^{\mathrm{PAB}} \leq \lambda^{\mathrm{PAB}} \leq \lambda_{\max}^{\mathrm{PAB}}$ for PAB. Both SMCs are assumed to be uniformly distributed random variables. Later in this paper, we show how these SMC uncertainty ranges are related to the common demand uncertainty range, a link that allows us to compare the two methods on a common footing.

A. Probabilistic Strategic Offers Under MP

Under MP, as described in [6], a generator can maximize its expected profit by submitting the strategic offer described in Table I, here denoted by $a_i^{\mathrm{MP}} = a_i^{\mathrm{MP-MAX}}$. The corresponding maximum expected value is denoted by $m_i^{\mathrm{MP-MAX}}$.

We note that the true incremental cost offer a_i^* is one of the strategic offers which maximize the expected profit under MP.

TABLE I
STRATEGIC OFFERS UNDER MP WITH UNIFORMLY RANDOM λ^{MP} SUCH THAT $\lambda_{\min}^{\mathrm{MP}} \leq \lambda^{\mathrm{MP}} \leq \lambda_{\max}^{\mathrm{MP}}$

Condition	Strategic Offer	Maximum E{Profit}
$a_i^* \leq \lambda_{\min}^{MP}$	$a_i^{MP-MAX} \leq \lambda_{\min}^{MP}$	$P_{gi}^{\max}(\frac{\lambda_{\max}^{MP} + \lambda_{\min}^{MP}}{2} - a_i^*)$
$\lambda_{\min}^{MP} \leq a_i^* \leq \lambda_{\max}^{MP}$	$a_i^{MP-MAX} = a_i^*$	$\frac{P_{gi}^{\max}(\lambda_{\max}^{MP} - a_i^*)^2}{2(\lambda_{\max}^{MP} - \lambda_{\min}^{MP})}$
$a_i^* \geq \lambda_{\max}^{MP}$	$a_i^{MP-MAX} \geq \lambda_{\max}^{MP}$	0

TABLE II
STRATEGIC OFFERS UNDER PAB WITH UNIFORMLY RANDOM λ^{PAB} SUCH THAT $\lambda_{\min}^{\mathrm{PAB}} \leq \lambda^{\mathrm{PAB}} \leq \lambda_{\max}^{\mathrm{PAB}}$

Condition	Strategic Offer	Max E{Profit}
$a_i^* \leq 2\lambda_{\min}^{PAB} - \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} = \lambda_{\min}^{PAB}$	$P_{gi}^{\max}(\lambda_{\min}^{PAB} - a_i^*)$
$2\lambda_{\min}^{PAB} - \lambda_{\max}^{PAB} \leq a_i^* \leq \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} = \frac{\lambda_{\max}^{PAB} + a_i^*}{2}$	$\frac{P_{gi}^{\max}(\lambda_{\max}^{PAB} - a_i^*)^2}{4(\lambda_{\max}^{PAB} - \lambda_{\min}^{PAB})}$
$a_i^* \geq \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} \geq \lambda_{\max}^{PAB}$	0

B. Probabilistic Strategic Offers Under PAB

Under PAB, the generator's strategic offer that maximizes its expected profit is described in Table II and is denoted by $a_i^{\mathrm{PAB}} = a_i^{\mathrm{PAB-MAX}}$. The corresponding maximum expected value is denoted by $m_i^{\mathrm{PAB-MAX}}$.

III. STANDARD DEVIATION OF PROFITS UNDER STRATEGIC OFFERS

When the profit of a generator is random as is the case here, the comparison of the two pricing methods should be based on both the expected value and on the standard deviation of the profit. The standard deviation provides a measure of the uncertainty of the expected value; a high standard deviation indicating that there is a high risk of deviating up or down with respect to the expected value.

If for MP we denote the randomly distributed profit under the strategic offer as $\mathrm{pr}_i^{\mathrm{MP}}(\lambda, a_i^{\mathrm{MP-MAX}})$, the *variance* of the profit is

$$\mathrm{var}_i^{\mathrm{MP-MAX}} = \frac{\int_{\lambda_{\min}^{\mathrm{MP}}}^{\lambda_{\max}^{\mathrm{MP}}} \left(pr_i^{\mathrm{MP}}\left(\lambda, a_i^{\mathrm{MP-MAX}}\right) - m_i^{\mathrm{MP-MAX}}\right)^2 d\lambda}{\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}}. \tag{9}$$

With the profit function specified in (8), we obtain

$$\mathrm{var}_i^{\mathrm{MP-MAX}} = \begin{cases} \sigma_{i\,\mathrm{MP-1}}^2, & \text{if } a_i^* \leq \lambda_{\min}^{\mathrm{MP}} \\ \sigma_{i\,\mathrm{MP-2}}^2, & \text{if } \lambda_{\min}^{\mathrm{MP}} \leq a_i^* \leq \lambda_{\max}^{\mathrm{MP}} \\ 0, & \text{if } a_i^* \geq \lambda_{\max}^{\mathrm{MP}} \end{cases} \tag{10}$$

where

$$\sigma_{i\,\mathrm{MP-1}}^2 = \frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}\right)^2}{12} \tag{11}$$

and,

$$\sigma_{i\,\mathrm{MP-2}}^2 = \frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda_{\max}^{\mathrm{MP}} - a_i^*\right)^3}{3\left(\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}\right)} \left(3\alpha^2 - 3\alpha + 1\right) \tag{12}$$

having defined $\alpha = (\lambda_{\max}^{\mathrm{MP}} - a_i^*)/(2(\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}))$.

Similarly, the variance of the profit under PAB is given by the integral

$$\mathrm{var}_i^{\mathrm{PAB-MAX}} = \frac{\int_{\lambda_{\min}^{\mathrm{PAB}}}^{\lambda_{\max}^{\mathrm{PAB}}} \left(\mathrm{pr}_i^{\mathrm{PAB}}\left(\lambda, a_i^{\mathrm{PAB-MAX}}\right) - m_i^{\mathrm{PAB-MAX}}\right)^2 d\lambda}{\lambda_{\max}^{\mathrm{PAB}} - \lambda_{\min}^{\mathrm{PAB}}} \tag{13}$$

whose solution is

$$\mathrm{var}_i^{\mathrm{PAB-MAX}} = \begin{cases} 0, & \text{if } a_i^* \le 2\lambda_{\min}^{\mathrm{PAB}} - \lambda_{\max}^{\mathrm{PAB}} \\ \sigma_i^2, & \text{if } 2\lambda_{\min}^{\mathrm{PAB}} - \lambda_{\max}^{\mathrm{PAB}} \le -a_i^* \le \lambda_{\max}^{\mathrm{PAB}} \\ 0, & \text{if } a_i^* \ge \lambda_{\max}^{\mathrm{PAB}} \end{cases} \tag{14}$$

where

$$\sigma_i^2 = \frac{\left(\lambda_{\max}^{\mathrm{PAB}} - a_i^*\right)^3 \left(P_{gi}^{\max}\right)^2 \left(\lambda_{\max}^{\mathrm{PAB}} - 2\lambda_{\min}^{\mathrm{PAB}} + a_i^*\right)^2}{32\left(\lambda_{\max}^{\mathrm{PAB}} - \lambda_{\min}^{\mathrm{PAB}}\right)^3}. \tag{15}$$

IV. Consumer Payments

In addition, to the expected values and standard deviations of the generator profits, a comparison of the two pricing methods should also look at the expected value of the consumer payments. Since we do not consider transmission losses or congestion, the consumer payment $\wp$ is the sum of the individual generator revenues, $\Re_i$

$$\wp = \sum_i \Re_i. \tag{16}$$

Therefore, the expected value of the total consumer payment is the sum of the individual expected generator revenues

$$E(\wp) = \sum_i E(\Re_i). \tag{17}$$

As shown in Part I, under PAB the revenue of generator i is the offered cost, that is

$$\begin{aligned}\Re_i^{\mathrm{PAB}} &= C_i\left(P_{gi}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right), a_i^{\mathrm{PAB}}\right) \\ &= u_i\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right) a_i^{\mathrm{PAB}} P_{gi}^{\max}\end{aligned} \tag{18}$$

where the expected revenue is

$$E\left(\Re_i^{\mathrm{PAB}}\right) = \frac{\int_{\lambda_{\min}^{\mathrm{PAB}}}^{\lambda_{\max}^{\mathrm{PAB}}} u_i\left(\lambda, a_i^{\mathrm{PAB}}\right) a_i^{\mathrm{PAB}} P_{gi}^{\max} d\lambda}{\lambda_{\max}^{\mathrm{PAB}} - \lambda_{\min}^{\mathrm{PAB}}}. \tag{19}$$

Using the strategic offer described in Table II and the unit commitment in (5), $E(\Re_i^{\mathrm{PAB}})$ becomes

$$E\left(\Re_i^{\mathrm{PAB}}\right) = \begin{cases} P_{gi}^{\max}\lambda_{\min}^{\mathrm{PAB}}, & \text{if } a_i^* \le 2\lambda_{\min}^{\mathrm{PAB}} - \lambda_{\max}^{\mathrm{PAB}} \\ \frac{P_{gi}^{\max}\left[\left(\lambda_{\max}^{\mathrm{PAB}}\right)^2 - (a_i^*)^2\right]}{4\left(\lambda_{\max}^{\mathrm{PAB}} - \lambda_{\min}^{\mathrm{PAB}}\right)}, & \text{if } 2\lambda_{\min}^{\mathrm{PAB}} - \lambda_{\max}^{\mathrm{PAB}} \le a_i^* \le \lambda_{\max}^{\mathrm{PAB}} \\ 0, & \text{if } a_i^* \ge \lambda_{\max}^{\mathrm{PAB}}. \end{cases} \tag{20}$$

Under MP, offering the true cost strategic offer, the revenue of unit i is

$$\Re_i^{\mathrm{MP}} = \lambda^{\mathrm{MP}} u\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right) P_{gi}^{\max}. \tag{21}$$

Then, the expected revenue is

$$E\left(\Re_i^{\mathrm{MP}}\right) = \begin{cases} \frac{P_{gi}^{\max}\left(\lambda_{\max}^{\mathrm{MP}} + \lambda_{\min}^{\mathrm{MP}}\right)}{2}, & \text{if } a_i^* \le \lambda_{\min}^{\mathrm{MP}} \\ \frac{P_{gi}^{\max}\left[\left(\lambda_{\max}^{\mathrm{MP}}\right)^2 - (a_i^*)^2\right]}{2\left(\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}\right)}, & \text{if } \lambda_{\min}^{\mathrm{MP}} \le a_i^* \le \lambda_{\max}^{\mathrm{MP}} \\ 0, & \text{if } a_i^* \ge \lambda_{\max}^{\mathrm{MP}}. \end{cases} \tag{22}$$

V. Market Behavior Comparison Between PAB and MP

We now characterize the market behavior under PAB and MP assuming that the generators submit the strategic probabilistic offers defined above subject to a predicted load that is random but known to lie inside the common range, $[P_d^{\min}, P_d^{\max}]$. First, we introduce the following theorem.

Theorem: Let the demand be uncertain but within the common known range $[P_d^{\min}, P_d^{\max}]$ for both MP and PAB. Let the associated SMCs be random and uniformly distributed within the uncertainty range $[\lambda_{\min}^{\mathrm{PAB}}, \lambda_{\max}^{\mathrm{PAB}}]$ under PAB, and within $[\lambda_{\min}^{\mathrm{MP}}, \lambda_{\max}^{\mathrm{MP}}]$ under MP. Under each pricing rule, let all generators use the respective expected-profit maximizing strategy. Then we have the following.

a) The SMC uncertainty ranges satisfy the relations

$$\lambda_{\max}^{\mathrm{PAB}} = \lambda_{\max}^{\mathrm{MP}} \quad \text{and} \quad \lambda_{\min}^{\mathrm{PAB}} = \frac{\lambda_{\min}^{\mathrm{MP}} + \lambda_{\max}^{\mathrm{PAB}}}{2}.$$

b) The expected profits of generator i under PAB and MP are equal, that is, $m_i^{\mathrm{MP-MAX}} = m_i^{\mathrm{PAB-MAX}}$.

c) The expected consumer payments (or total generator revenues) under PAB and MP are equal, that is

$$\sum_i E\left\{\Re_i^{\mathrm{MP}}\right\} = \sum_i E\left\{\Re_i^{\mathrm{PAB}}\right\}.$$

d) The variances of the generator profits under MP are greater than or equal to the variances under PAB, that is, $\mathrm{var}_i^{\mathrm{MP-MAX}} \ge \mathrm{var}_i^{\mathrm{PAB-MAX}}$.

e) The consumer payment under MP lies within an uncertainty range which contains the range of uncertainty of the consumer payment under PAB.

This theorem is proven in the Appendix.

- Conclusion (a) of this theorem is fundamental and makes it possible to relate the uncertainty ranges $\lambda_{\min}^{\mathrm{MP}} \le \lambda^{\mathrm{MP}} \le \lambda_{\max}^{\mathrm{MP}}$ for MP and $\lambda_{\min}^{\mathrm{PAB}} \le \lambda^{\mathrm{PAB}} \le \lambda_{\max}^{\mathrm{PAB}}$ for PAB to the common demand uncertainty range, $[P_d^{\min}, P_d^{\max}]$. This relationship is illustrated in Fig. 1. One obvious observation is that for a common demand uncertainty range, the corresponding SMC uncertainty range under MP is twice the size of the SMC uncertainty range under PAB.
- Conclusions (b) and (c) of the theorem indicate that irrespective of the pricing method, consumers pay and generators collect the same amount, on the average.
- Conclusions (d) and (e) of the theorem say that although the expected generator profits and expected consumer payment are independent of the pricing method, the uncertainty in these expected values as defined by the variance

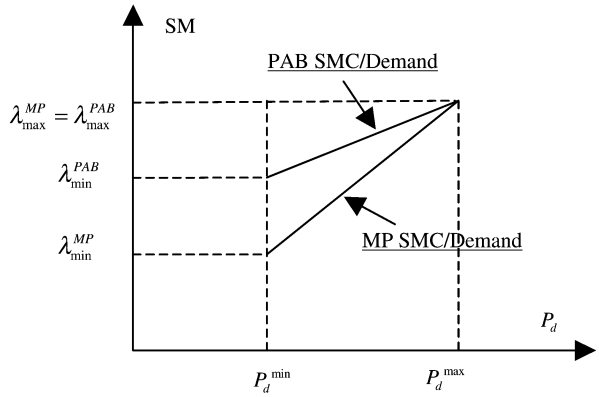

Fig. 1. SMC versus demand behavior under PAB and MP subject to a same demand uncertainty.

TABLE III
COST DATA

Unit number	True Incremental Cost a_i^* ($/MWh)	$P_{gi}^{\max}$ (MW)
400	40.0	50
401	40.4	50
…	…	…
400+i	40+0.4*i	50
…	…	…
450	60.0	50

TABLE IV
SMC UNCERTAINTY RANGES ($/MWh)

MP	$\lambda_{\min}^{MP}$	40
	$\lambda_{\max}^{MP}$	56
PAB	$\lambda_{\min}^{PAB}$	48
	$\lambda_{\max}^{PAB}$	56

of the generator profits or by the uncertainty range of the consumer payment is worse under MP than under PAB.

VI. NUMERICAL EXAMPLES

We approximate a perfect market by a large number of units each offering a maximum output of 50 MW, ordered in a monotonically increasing sequence of a_i^*. Table III lists units 400 through 450, one of whose true incremental costs defines the SMC.

We suppose that the demand is uniformly uncertain in the range [20 000 22 000] MW. As depicted in Fig. 1, we can obtain the SMC uncertainty ranges under PAB and MP as shown in Table IV.

Table V shows the strategic offers, the expected values, and the standard deviations of the unit profits under PAB and MP. The different strategic offers submitted by the generators are clearly seen by comparing columns 2 and 5. Columns 3 and 6 confirm that the two pricing rules yield the same expected profits when the generating units submit strategic offers. Finally, columns 4 and 7 show that the standard deviations of the generator profits are always lower under PAB than under MP, and for some units significantly lower.

TABLE V
STRATEGIC OFFERS AND PROFIT RESULTS UNDER PAB AND MP

	MP			PAB		
Unit	Strategic Offer $/MWh	E{Pr} $/h	Standard Dev. $/h	Strategic Offer $/MWh	E{Pr} $/h	Standard Dev. $/h
400	40.0	400	236.6	48.0	400	0
401	40.4	380.5	235.8	48.2	380.5	60.2
402	40.8	361.5	234.3	48.4	361.5	81.9
…	…		…	…		…
410	44.0	226.8	202.0	48.0	226.8	128.83
411	44.4	212.2	196.1	50.2	212.2	128.49
412	44.8	198.5	189.8	50.4	198.5	127.4
…	…		…	…		…
430	52.0	26.8	55.1	54.0	26.8	44.31
431	52.4	21.95	47.9	54.2	21.95	38.65
432	52.8	17.56	41.1	54.4	17.56	33.11
…	…		…	…		…
438	55.2	1.46	6.83	55.6	1.46	5.21
439	55.6	0.49	3.08	55.8	0.49	2.15
440	56	0	0	56	0	0
…	…		…	…		…
450	60.0	0	0	60.0	0	0

TABLE VI
EXPECTED CONSUMER PAYMENT AND STANDARD DEVIATION

	Expected Consumer Payment $/h	Calculated Standard deviation $/h	Uncertainty Range $/h
MP	1,010,800	127,810	[8.0, 12.3]*10^5
PAB	1,010,800	30,829	[9.6, 10.6]*10^5

TABLE VII
EXPECTED AVERAGE PRICE PAID BY CONSUMERS AND STANDARD DEVIATION

	Expected Average Price $/MWh	Calculated Standard deviation $/MWh
MP	48	4.73
PAB	48.13	0.11

The first column of numbers in Table VI reveals that, as predicted by the theory, the expected consumer payments are the same under MP and PAB. The results also confirm that the standard deviation under MP is greater than that under PAB, in this case about four times worse. The table also confirms that the uncertainty range under MP is larger and contains the uncertainty range under PAB.

In terms of average price paid by the consumers, Table VII shows that consumers pay slightly more on the average for PAB compared to MP, however the standard deviation is again significantly worse under MP, that is 4.73 $/MWh versus 0.11 $/MWh. This last measure cannot be derived theoretically and can only be determined by simulation.

A second numerical example analyzed takes 26 generating unit types from the IEEE RTS-96 [11] system, each divided into four blocks of energy for a total of 104 energy blocks of different lengths and incremental costs. The five hydro units are fully committed. The data characterizing these units is given in Table VIII.

TABLE VIII
INCREMENTAL COST DATA OF UNIT GROUPS BY BLOCKS

Unit Group	Offer	True IC, a_i^* ($/MWh)	$P_{gi}^{\max}$ (MW)
U12	1	23.412	2.4
	2	23.759	3.6
	3	26.836	3.6
	4	30.404	2.4
U20	1	29.577	15.8
	2	30.417	0.2
	3	42.816	2
	4	43.281	2
U76	1	11.458	15.2
	2	11.959	22.8
	3	13.894	22.8
	4	15.973	15.2
U100	1	18.605	25
	2	20.028	25
	3	21.666	30
	4	22.717	20
U155	1	9.918	54.25
	2	10.249	38.75
	3	10.68	31
	4	11.257	31
U197	1	19.2	68.95
	2	20.316	49.25
	3	21.218	39.4
	4	22.126	39.4
U350	1	10.082	140
	2	10.675	87.5
	3	11.093	52.5
	4	11.722	70
U400	1	5.308	100
	2	5.379	100
	3	5.526	120
	4	5.663	80

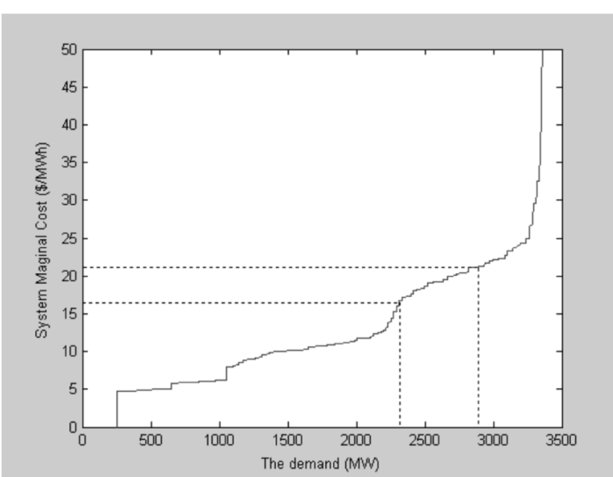

Fig. 2. System marginal cost versus total demand for RTS system.

This system is made up of 5 units of U12, 4 of U20, 4 of U76, 3 of U100, 4 of U155, 3 of U197, 1 of U350, and 2 of U400. To be able to differentiate more clearly among otherwise identical generating units, their incremental costs have been adjusted by small amounts within ±10%. For the above data, Fig. 2 shows the corresponding System Marginal Cost (SMC) versus total demand. When the demand uncertainty lies between 2308.8 and 2968.2 MW, and the market follows the strategic offer behavior derived in this paper, the resulting range of SMC under the two pricing methods is summarized in Table IX.

TABLE IX
SMC UNCERTAINTY RANGES FOR IEEE RTS-96 DATA ($/MWh)

MP	$\lambda_{\min}^{MP}$	16.74
	$\lambda_{\max}^{MP}$	21.67
PAB	$\lambda_{\min}^{PAB}$	19.21
	$\lambda_{\max}^{PAB}$	21.67

TABLE X
STRATEGIC OFFERS AND PROFITS UNDER PAB AND MP: SAMPLE RESULTS OF IEEE RTS-96 DATA

Unit and Block No.		MP			PAB		
		Strategic Offer $/MWh	E{Pr} $/h	Std. Dev. $/h	Strategic Offer $/MWh	E{Pr} $/h	Std. Dev. $/h
U12	1	18.73	2.48	2.38	20.20	2.41	1.64
	2	19.01	3.04	3.26	20.34	3.26	2.23
	3	21.47	0.03	0.15	21.57	0.02	0.08
	4	24.32	0	0	24.32	0	0
U76	1	12.60	104.00	23.38	19.21	100.35	0
	2	13.16	143.42	32.07	19.21	137.94	0
	3	15.28	94.91	32.07	19.21	89.43	0
	4	17.57	29.44	19.87	19.62	26.69	10.88
U100	1	16.74	67.53	35.17	19.21	61.52	0
	2	18.03	38.93	30.06	19.85	37.97	16.91
	3	19.50	17.06	21.72	20.58	16.59	16.25
	4	20.45	3.72	7.17	21.06	3.57	5.55
U197	1	17.28	150.69	94.52	19.47	145.46	28.88
	2	18.28	66.53	55.99	19.98	66.33	33.52
	3	19.10	30.95	34.65	20.38	34.17	23.72
	4	19.91	14.82	22.1	20.79	16.06	17.22

TABLE XI
EXPECTED AVERAGE PRICE PAID BY CONSUMERS AND STANDARD DEVIATION

	Expected Average Price $/MWh	Calculated Standard deviation $/MWh
MP	19.21	1.478
PAB	19.33	0.099

Table X shows some selected statistical individual profit results obtained by running a large number of load levels within the given uncertainty range. Note that the expected profits between MP and PAB are slightly different because these are estimated numerically. What is consistent with the earlier example and the theory is that the standard deviations of the profits remain considerably higher under MP in comparison with PAB. Table XI confirms these conclusions with respect to the numerically estimated standard deviations of the average price paid by the customers.

VII. CONCLUSION

To be able to compare the PAB and MP pricing rules under common system demand conditions, we assume that the demand is a random variable over a known range, and that the resulting system marginal costs are uniformly distributed random variables. We also assume that all generators use the strategic offers derived in Part I that maximize the expected value of each individual profit. This paper then establishes a theoretical relation between the common system demand and the system marginal cost for both PAB and MP, thereby providing a link between the SMC uncertainty ranges under both pricing methods. This fundamental step allows us to reach the following conclusions.

- The expected value of the profit of an arbitrary generator is the same under PAB and MP.
- The expected value of the consumer payment is the same under PAB and MP.
- The standard deviation of the profit of an arbitrary generator is greater under MP than under PAB.
- The range of uncertainty of the consumer payment under MP is larger than and contains the corresponding range of uncertainty under PAB.

Numerical simulations confirm these comparative statements, suggesting that, under perfect competition, MP is financially riskier than PAB for both consumers and suppliers.

Notwithstanding these comparative results, we must be careful in extrapolating them to systems where some generating units can exercise market power to control the price and gain a profit advantage. A comparative quantitative study of PAB and MP under market power conditions will inevitably require numerical simulation and assumptions about risk tolerance on the part of the competing agents. Still, the results obtained here under perfect market assumptions shed new light into the MP versus PAB debate, bringing up some new significant questions that need further study in real markets, primarily: Is there less price and profit uncertainty under PAB compared to MP? If the risks of making less profit are higher under PAB, are agents less likely to game under PAB?

APPENDIX

In this Appendix, we prove the Theorem.

A. Proof of Part (a)

Under MP, the SMC/demand or λ^{MP} versus P_d curve is characterized by

$$P_d = \sum_i u_i \left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP-MAX}}\right) P_{gi}^{\max}. \tag{23}$$

As derived in Part I, the strategic offer that maximizes the expected profit under MP is $a_i^{\mathrm{MP-MAX}} = a_i^*$, the true incremental cost. In (23), therefore, all units are scheduled on in increasing order of a_i^*.

Under PAB, the λ^{PAB} versus P_d curve is characterized by

$$P_d = \sum_i u_i \left(\lambda^{\mathrm{PAB}}, a_i^{PAB-MAX}\right) P_{gi}^{\max}. \tag{24}$$

By examining the nature of the strategic PAB offer, $a_i^{\mathrm{PAB-MAX}}$, in Table II, we observe that in (24) the generating units are also scheduled on in increasing order of the cost parameter, a_i^*.

Therefore, for the same demand, P_d, both PAB and MP yield identical generation schedules.

Suppose that unit j is the most expensive of the scheduled on units when supplying $P_d^{\max}$. Then, under MP

$$\lambda^{\mathrm{MP}} = a_j^* \tag{25}$$

while under PAB

$$\lambda^{\mathrm{PAB}} = \frac{a_j^* + \lambda_{\max}^{\mathrm{PAB}}}{2}. \tag{26}$$

However, since $P_d^{\max}$ is the highest value in the demand uncertainty interval, the corresponding λ^{PAB} must also be the maximum in the uncertainty interval $[\lambda_{\min}^{\mathrm{PAB}}, \lambda_{\max}^{\mathrm{PAB}}]$; in other words

$$\lambda_{\max}^{\mathrm{PAB}} = \frac{a_j^* + \lambda_{\max}^{\mathrm{PAB}}}{2}. \tag{27}$$

From (26) and (27), it follows that $\lambda_{\max}^{\mathrm{PAB}} = a_j^*$.

Similarly, the λ^{MP} corresponding to $P_d^{\max}$ must be the maximum in the uncertainty interval $[\lambda_{\min}^{\mathrm{MP}}, \lambda_{\max}^{\mathrm{MP}}]$, in other words, $\lambda^{\mathrm{MP}} = \lambda_{\max}^{\mathrm{MP}} = a_j^*$. It therefore follows that

$$\lambda_{\max}^{\mathrm{MP}} = \lambda_{\max}^{\mathrm{PAB}} \triangleq \lambda^{\max} \tag{28}$$

thus, proving the first statement of part (a).

Similarly, assume that unit is the most expensive of the scheduled on units when the demand is $P_d^{\min}$. Then, under MP, the value of λ^{MP} must correspond to the minimum SMC in the interval $[\lambda_{\min}^{\mathrm{MP}}, \lambda_{\max}^{\mathrm{MP}}]$. Then

$$\lambda^{\mathrm{MP}} = \lambda_{\min}^{\mathrm{MP}} = a_k^*. \tag{29}$$

Similarly, under PAB

$$\lambda^{\mathrm{PAB}} = \lambda_{\min}^{\mathrm{PAB}} = \frac{a_k^* + \lambda_{\max}^{\mathrm{PAB}}}{2}. \tag{30}$$

Therefore

$$\lambda_{\min}^{\mathrm{PAB}} = \frac{\lambda_{\min}^{\mathrm{MP}} + \lambda^{\max}}{2} \tag{31}$$

which completes the Proof of (a).

B. Proof of Part (b)

Using part (a) of the Theorem, the maximum expected profit under PAB in Table II can be rewritten as

$$m_i^{\mathrm{PAB-MAX}} = \begin{cases} \left(\frac{\lambda_{\min}^{\mathrm{MP}}+\lambda^{\max}}{2} - a_i^*\right) P_{gi}^{\max}, & \text{if } a_i^* \le \lambda_{\min}^{\mathrm{MP}} \\ \frac{(\lambda^{\max}-a_i^*)^2 P_{gi}^{\max}}{2\left(\lambda^{\max}-\lambda_{\min}^{\mathrm{MP}}\right)}, & \text{if } \lambda_{\min}^{\mathrm{MP}} \le a_i^* \le \lambda^{\max} \\ 0, & \text{if } a_i^* \ge \lambda^{\max} \end{cases} \tag{32}$$

which is identical to the maximum expected profit under MP.

Therefore, if all independent units use the maximum expected-profit offer strategy, they obtain the same expected profit under both PAB and MP. This proves part (b) of the Theorem.

C. Proof of Part (c)

Using part (a) and comparing (20) with (22) we obtain

$$E\left(\Re_i^{\mathrm{PAB}}\right) = E\left(\Re_i^{\mathrm{MP}}\right). \tag{33}$$

It then follows that

$$E(\wp^{\mathrm{PAB}}) = E(\wp^{\mathrm{MP}}). \tag{34}$$

Equation (34) shows that the expected value of the consumer payment is identical under PAB and MP. This completes part (c).

D. Proof of Part (d)

Using part (a) of the Theorem, (14) reduces to

$$\mathrm{var}_i^{\mathrm{PAB}} = \begin{cases} 0, & \text{if } a_i^* \le \lambda_{\min}^{\mathrm{MP}} \\ \sigma_i^2, & \text{if } \lambda_{\min}^{\mathrm{MP}} \le a_i^* \le \lambda^{\max} \\ 0, & \text{if } a_i^* \ge \lambda^{\max} \end{cases} \tag{35}$$

and (15) becomes

$$\sigma_i^2 = \frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda^{\max} - a_i^*\right)^3 \left(a_i^* - \lambda_{\min}^{\mathrm{MP}}\right)^2}{4\left(\lambda^{\max} - \lambda_{\min}^{\mathrm{MP}}\right)^3}. \tag{36}$$

Comparing (36) with (12)

$$\begin{aligned} \sigma_i^2 - \sigma_{i\,\mathrm{MP}-2}^2 &= \frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda^{\max} - a_i^*\right)^3 \left(a_i^* - \lambda_{\min}^{\mathrm{MP}}\right)^2}{4\left(\lambda_{\max}^{\mathrm{MP}} - \lambda_{\min}^{\mathrm{MP}}\right)^3} \\ &\quad - \frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda^{\max} - a_i^*\right)^3}{3\left(\lambda^{\max} - \lambda_{\min}^{\mathrm{MP}}\right)} (3\alpha^2 - 3\alpha + 1) \\ &= -\frac{\left(P_{gi}^{\max}\right)^2 \left(\lambda^{\max} - a_i^*\right)^3}{12\left(\lambda^{\max} - \lambda_{\min}^{\mathrm{MP}}\right)}. \\ &< 0. \end{aligned} \tag{37}$$

If, in addition, we examine (14) and (10), it is clear that the variance of the random profit is lower under PAB than under MP. This completes part (d) of the Theorem.

E. Proof of Part (e)

An approximation of the variance of the consumer payment can be analytically found by computing its extreme values as follows.

When $P_d = P_d^{\min}$, the payment under PAB is

$$\begin{aligned} \wp^{\mathrm{PAB}}\left(\lambda_{\min}^{\mathrm{PAB}}\right) &= \sum_i \Re_i^{\mathrm{PAB}}\left(\lambda_{\min}^{\mathrm{PAB}}\right) \\ &= \lambda_{\min}^{\mathrm{PAB}} \sum_{i \in \mathrm{Min}} P_{gi}^{\max} \\ &= \lambda_{\min}^{\mathrm{PAB}} P_d^{\min} \end{aligned} \tag{38}$$

where Min is a set of generators with offer $a_i^{\mathrm{PAB-MAX}} = \lambda_{\min}^{\mathrm{PAB}}$.

If $P_d = P_d^{\max}$ and Max denotes the set of generators which offer $a_i^{\mathrm{PAB-MAX}} \le \lambda^{\max}$, then

$$\begin{aligned} \wp^{\mathrm{PAB}}(\lambda^{\max}) &= \sum_i \Re_i^{\mathrm{PAB}}(\lambda^{\max}) \\ &= \lambda_{\min}^{\mathrm{PAB}} P_d^{\min} + \sum_{\substack{i \in \mathrm{Max} \\ i \notin \mathrm{Min}}} a_i{}^{\mathrm{PAB-MAX}} P_{gi}^{\max} \\ &< \lambda_{\min}^{\mathrm{PAB}} P_d^{\min} + \lambda^{\max}\left(P_d^{\max} - P_d^{\min}\right) \\ &< \lambda^{\max} P_d^{\max}. \end{aligned} \tag{39}$$

Under MP

$$\wp^{\mathrm{MP}}(\lambda_{\min}^{\mathrm{MP}}) = \lambda_{\min}^{\mathrm{MP}} P_d^{\min} \tag{40}$$

$$\wp^{\mathrm{MP}}(\lambda^{\max}) = \lambda^{\max} P_d^{\max}. \tag{41}$$

It is clear from (38)–(41) that $\wp^{\mathrm{PAB}}(\lambda_{\min}^{\mathrm{PAB}}) > \wp^{\mathrm{MP}}(\lambda_{\min}^{\mathrm{MP}})$ and that $\wp^{\mathrm{PAB}}(\lambda_{\max}^{\mathrm{PAB}}) < \wp^{\mathrm{MP}}(\lambda_{\max}^{\mathrm{MP}})$. This result indicates that the consumer payments under PAB lie in a narrower range contained inside the range of consumer payments under MP. This proves part (e) of the Theorem.

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