Pay-as-Bid versus Marginal Pricing—Part I: Strategic Generator Offers

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***Abstract*—As the arguments for and against the use of pay-as-bid (PAB) or marginal pricing (MP) in electricity pools tend to be qualitative, we compare the quantitative behavior of the two markets assuming that generators submit the best strategic offers that correspond to the specified pricing method. In Part I of this two-part study, assuming that the system marginal costs for PAB and MP are random with known probability density functions, we develop generator strategic offers by maximizing the corresponding expected values of the generator profits over the offer parameters. In Part II relations are established between the SMCs for each market type and a common random demand, thus allowing the two markets to be compared through the expected values and variances of the individual generation profits and of the consumer payments. This comparison demonstrates both theoretically and through simulation that: 1) the expected values of the individual generator profits as well as of the consumer payments are the same under MP and PAB and 2) the variances of the individual generator profits and of the consumer payments however are larger under MP than under PAB. The primary conclusion is then that although MP and PAB yield identical expected generator profits and consumer payments, the risk of not meeting these expected values is greater under MP than under PAB.**

***Index Terms*—Expected profit, marginal pricing, pay-as-bid, perfectly competitive markets, strategic offers, system marginal cost, uncertainty.**

I. Introduction

WHETHER "pay as bid" (PAB) pricing can or should replace the more common marginal pricing (MP) in electricity pool markets is the subject of an on-going spirited debate [1]–[5]. Under both approaches, a centralized system operator (SO) schedules the system generation by minimizing the combined cost of all the generation offers. However, under marginal pricing, generators are remunerated at a rate equal to the system marginal cost (SMC), while under PAB pricing the generators are paid the exact cost that they quote to the SO. Thus, to make a profit under PAB, generators must submit offers greater than their true costs, while under MP generators may still earn a profit even if they offer to generate at their true costs. In terms of auction theory, PAB is a variation of sealed first-price auction while MP is a variation of sealed second-price auction [9].

One argument against MP is that generators with market power may "game" by submitting offers above their true cost in order to increase their share of the profits. Then, a market that minimizes the total offered cost does not necessarily maximize social welfare. Under PAB however, whether generators game or not, the resulting market equilibrium always minimizes the total consumer payment. A further argument expressed in favor of PAB pricing is that since it is based on an average rather than a marginal price, it may be less volatile to gaming. Critics of PAB contend that only under marginal pricing does the market price reflect a surplus or deficit of generation capacity, thus sending a "correct" long-term economic signal to potential investors. This economic argument is very persuasive and has resulted in the implementation of marginal pricing as the dominant scheme in many types of markets. One recent exception is the case of the electricity market in England and Wales where a scheme based on bilateral agreements which are essentially PAB has been adopted recently.

Nonetheless, it has been maintained that under PAB, generators would quickly learn how to adjust their offers so as to attain profits that would equal or exceed those profits obtained under MP [5]. A Blue Ribbon Panel [3] contends that methods such as PAB pricing "would do consumers more harm than good" since, in attempting to earn acceptable profits, the average cost of electricity would be driven higher.

Since the debate between PAB and MP methods is rooted on qualitative or numerical simulation arguments [11]–[13], we have conducted a rigorous quantitative comparison of the behavior of electricity markets under both pricing approaches. This comparison recognizes that generators will adapt their profit-maximizing strategic offers to the specific pricing scheme, thereby altering the resulting market behavior in terms of generation schedules, system and marginal costs, and consumer payments.

The comparative study assumes a perfect market where the system marginal cost is unaffected by any individual generator offer. This assumption could be eventually relaxed to represent oligopolies where some competing entities possess market power. The difficulty with "imperfect" market models however is that comparative statements between PAB and MP cannot be made without extensive numerical simulations and, especially, without specific assumptions about how individual competitors respond to risk. The perfect market assumption is a compromise that allows us to develop general comparisons of a quantitative nature that do not rely on market simulations and risk tolerance assumptions. One significant characteristic of a perfect market is that individual generators have no advantage in gaming and, in fact, were a generator to game, it would increase its risk of making less profit.

The results are presented in a two-part paper. In this first part, we develop the "best" generator offers for both PAB and MP

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recognizing that since the system demand cannot be exactly predicted, the forecast of the system marginal costs (SMC) are also uncertain. Part I then develops strategic generator offers that maximize the expected value of their profit under PAB and MP. It assumed that the SMCs are random variables with known probability density functions that are generally different from one pricing scheme to the other, and that these functions can be obtained from historical SMC data. The generator strategic offers and the corresponding best expected profits are then derived in analytic form for the PAB and MP pricing rules, clearly indicating a number of fundamental differences.

Since the strategic offers vary according to the pricing method, the relation between the system marginal cost and the system demand also differs between PAB and MP. As a result, a fair comparison of the behavior of the two markets cannot be made at this stage. To do this in an equitable manner, both markets must operate under common demand conditions, in other words, under the same demand probability distribution for both pricing methods.

Part II [10] therefore first develops analytic expressions relating the SMCs under PAB and MP to the common system demand. When the generators submit the strategic offers derived in Part I, these relations then permit us to compare the behavior of the two markets demonstrating both theoretically and through simulation that: 1) The expected value of the individual generator profits and the consumer payments are the same under MP and PAB and 2) the variances of the individual generator profits and the consumer payments are however larger under MP than under PAB.

The main conclusion of Part II is then that although MP and PAB yield identical expected generator profits and consumer payments, the risk of not meeting these expected values is greater under MP than under PAB.

II. Market Modelling Assumptions

In an electricity pool, under either PAB or MP pricing, each generator submits an offer to sell power. A centralized entity like the system operator (SO) is responsible for scheduling the generators (unit commitment), and for dispatching the power of those units that are scheduled on (economic dispatch). A fundamental assumption is that, under either MP or PAB, a power pool schedules generation by minimizing the total offered cost. In its most basic form, this problem takes the following form:

$$\min_{u,P_g} C = \sum_i C_i(P_{gi}, a_i). \tag{1}$$

The cost offered by generator i, C_i, for simplicity, is here characterized by only one parameter a_i, that is

$$C_i(P_{gi}, a_i) = a_i P_{gi}. \tag{2}$$

However, as discussed in Appendix B, a more general formulation of the cost offer can be considered consisting of several different blocks. As shown in Appendix B, the strategic offers derived for MP and PAB can be applied to each block as if it were an independent generating unit. As a result, the single block offer assumption can be made without loss of generality.

The generation output P_{gi} is restricted to lie between two technical limits

$$u_i P_{gi}^{\min} \le P_{gi} \le u_i P_{gi}^{\max} \tag{3}$$

where the binary $0/1$ variable u_i denotes whether the corresponding unit is scheduled off or on.

The offer parameter a_i may differ from the true value a_i^*, the difference reflecting the possibility that the ith generator offers to produce power at a cost other that its true cost as an intentional gaming strategy aimed at increasing its profit. In fact, under PAB, the offer a_i must be different from its true value if a generator is to make a profit.

In addition, as part of the offer, a generating unit could submit upper and lower bounds that differ from their true values, however to simplify the presentation, we assume that the offer is based on the true generation limits [7].

Under both PAB and MP markets, the objective function (1) is subject to the individual generation limits in (3), as well as to the power balance[1] between the total generation and the inelastic demand, P_d

$$\sum_i P_{gi} = P_d(\lambda). \tag{4}$$

It is important to re-emphasize that irrespective of whether marginal or PAB pricing is used, the market is cleared by solving the same type of optimization problem defined by (1)–(4).

The system marginal cost, dC/dP_d, is derived from the common market clearing procedure (1)–(4), and is equal to the Lagrange multiplier associated with the power balance (4), to which we assign the symbol λ. We note that under MP, λ becomes the price of electricity charged to the consumers and paid to the generators. Under PAB, however, the price of electricity charged to the consumers is the average cost of the scheduled generators, which is not necessarily equal to the SMC.

Notwithstanding this common market clearing approach, since the strategic offers, a_i, may generally differ from one pricing method to the other, the corresponding generation schedule, costs, profits, consumer payment and system marginal cost may also differ between MP and PAB.

III. Generation Scheduling Solution

For both PAB and MP, in a perfect market, the unit commitment solution of the market clearing optimization problem (1)–(4) says that a unit is scheduled on by the SO if the SMC, λ, is greater than or equal to the offered incremental cost, a_i. Alternatively if the system marginal cost is less than a_i, the unit is scheduled off [8]

$$u_i(\lambda, a_i) = \begin{cases} 1, & \text{if } \lambda \ge a_i \\ 0, & \text{if } \lambda < a_i \end{cases}. \tag{5}$$

[1]This model assumes a single price for energy as is the case in many real markets where transmission losses and congestion are accounted for through separate uplift charges.

In addition, if $\lambda > a_i$, the generator is on and is dispatched by the SO at its maximum output. Thus, for all units, except those with $\lambda = a_i$, the generation output is

$$P_{gi}(\lambda, a_i) = u_i(\lambda, a_i) P_{gi}^{\max}. \tag{6}$$

The units whose offers a_i are equal to the system marginal cost, λ, are called the marginal generators. These units operate somewhere between their upper and lower limits in order to balance the generation and demand. However, since under the perfect market assumption the maximum output of any generator is negligible compared to the system demand, the marginal generators can also be assumed to operate at their maximum output with negligible error.

Then, for both PAB and MP, the output of any unit i now takes the form of (6), while its offered cost becomes

$$C_i(P_{gi}(\lambda, a_i), a_i) = u_i(\lambda, a_i) a_i P_{gi}^{\max}. \tag{7}$$

The SMC under PAB is denoted by $\lambda = \lambda^{\mathrm{PAB}}$, a value that, for the same demand, P_d, may generally differ from the SMC under marginal pricing, $\lambda = \lambda^{\mathrm{MP}}$. Similarly, the offers under PAB and MP, respectively, a_i^{PAB} and a_i^{MP}, may differ. In Part II [10], we show that, for identical demand conditions, a relationship exists between the true SMCs, λ^{PAB} and λ^{MP}, which allows us to compare the two pricing rules on a common basis.

Under PAB, the profit of unit i is given by the following:

$$\begin{aligned} pr_i^{\mathrm{PAB}}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right) &= C_i\left(P_{gi}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right), a_i^{\mathrm{PAB}}\right) \\ &\quad - C_i\left(P_{gi}\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right), a_i^*\right) \\ &= u_i\left(\lambda^{\mathrm{PAB}}, a_i^{\mathrm{PAB}}\right)\left(a_i^{\mathrm{PAB}} - a_i^*\right) P_{gi}^{\max} \end{aligned} \tag{8}$$

while under MP, the profit of unit i is

$$\begin{aligned} pr_i^{\mathrm{MP}}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right) &= \lambda^{\mathrm{MP}} P_{gi}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right) \\ &\quad - C_i\left(P_{gi}\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right), a_i^*\right) \\ &= u_i\left(\lambda^{\mathrm{MP}}, a_i^{\mathrm{MP}}\right)\left(\lambda^{\mathrm{MP}} - a_i^*\right) P_{gi}^{\max}. \end{aligned} \tag{9}$$

In both (8) and (9) the profit is defined by the revenue minus the true cost, but the resulting functions are different.

IV. Strategic Offers With Uncertain Information

If we recognize that the system load can only be predicted with some degree of uncertainty, then the corresponding SMCs will also be uncertain, as will the generator profits. One way to quantify a randomly varying profit is through its expected value and standard deviation, in which case the probabilistic strategic goal of a generator is to maximize the expected value of its profit.

We now assume that the corresponding SMCs have known probability density functions, $f^{\mathrm{MP}}(\lambda^{\mathrm{MP}})$ and $f^{\mathrm{PAB}}(\lambda^{\mathrm{PAB}})$, both derived from publicly available historical data. Moreover, these probability density functions cannot be influenced by any single individual offer.

TABLE I
Expected Profit Under MP With Uniformly Random λ^{MP} Such That $\lambda_{\min}^{\mathrm{MP}} \le \lambda^{\mathrm{MP}} \le \lambda_{\max}^{\mathrm{MP}}$

Cases	Condition	Expected Profit, $m_i^{MP}(a_i^{MP})$
A	$a_i^{MP} \le \lambda_{\min}^{MP}$	$P_{gi}^{\max}\left(\dfrac{\lambda_{\max}^{MP}+\lambda_{\min}^{MP}}{2} - a_i^*\right)$
B	$\lambda_{\min}^{MP} \le a_i^{MP} \le \lambda_{\max}^{MP}$	$\dfrac{P_{gi}^{\max}(\lambda_{\max}^{MP} - a_i^{MP})}{\lambda_{\max}^{MP} - \lambda_{\min}^{MP}}\left(\dfrac{(\lambda_{\max}^{MP}+a_i^{MP})}{2} - a_i^*\right)$
C	$a_i^{MP} \ge \lambda_{\max}^{MP}$	0

We additionally assume that the SMC in both pricing methods is uniformly distributed[2] within the known range $[\lambda_{\min}^{\mathrm{MP}}, \lambda_{\max}^{\mathrm{MP}}]$ for MP and the possibly different but known range $[\lambda_{\min}^{\mathrm{PAB}}, \lambda_{\max}^{\mathrm{PAB}}]$ for PAB. In general, a uniformly distributed random variable λ over the interval $[\lambda_{\min}, \lambda_{\max}]$ has the probability density distribution, $f(\lambda)$, where

$$f(\lambda) = \frac{1}{\lambda_{\max} - \lambda_{\min}}. \tag{10}$$

Then, the expected value of the profit of this random variable $pr(\lambda)$ is given by

$$m = \frac{1}{\lambda_{\max} - \lambda_{\min}} \int_{\lambda_{\min}}^{\lambda_{\max}} pr(\lambda)\, d\lambda \tag{11}$$

while the variance is found from the following relation:

$$\sigma^2 = \frac{1}{\lambda_{\max} - \lambda_{\min}} \int_{\lambda_{\min}}^{\lambda_{\max}} (pr(\lambda) - m)^2\, d\lambda. \tag{12}$$

The variance σ^2 is analyzed in Part II of this paper [10], where the two pricing methods are compared under common demand conditions.

A. Strategic Offer Under MP

Under MP, the expected value of the profit of unit i for a given offer parameter, a_i^{MP}, is found by solving (11) over the known uncertainty range $[\lambda_{\min}^{\mathrm{MP}}, \lambda_{\max}^{\mathrm{MP}}]$ using relations (9) and (5) to define the profit function. The resulting expected profit function, denoted here by $m_i^{\mathrm{MP}}(a_i^{\mathrm{MP}})$, is readily found analytically as summarized in Table I for three possible ranges of the offer a_i^{MP}.

The probabilistic strategic offer of unit i under MP that maximizes its expected profit is now defined by the following:

$$\max_{a_i^{\mathrm{MP}}} \left\{ m_i^{\mathrm{MP}}\left(a_i^{\mathrm{MP}}\right)\right\}. \tag{13}$$

The value of a_i^{MP} that solves (13) and the corresponding maximum expected profit, denoted respectively by $a_i^{\mathrm{MP-MAX}}$ and by $m_i^{\mathrm{MP-MAX}} = m_i^{\mathrm{MP}}(a_i^{\mathrm{MP-MAX}})$, can be solved analytically as shown in Table II (see Appendix A).

[2]Other SMC probability distributions such as normal are possible but less tractable analytically. The maximum expected profit strategy as well as the maximum expected profit and its variance would then have to be found numerically by searching over the offer parameters.

TABLE II
STRATEGIC OFFERS UNDER MP WITH UNIFORMLY RANDOM λ^{MP} SUCH THAT $\lambda_{\min}^{\mathrm{MP}} \leq \lambda^{\mathrm{MP}} \leq \lambda_{\max}^{\mathrm{MP}}$

Condition	Strategic Offer	Maximum E{Profit}
$a_i^* \leq \lambda_{\min}^{MP}$	$a_i^{MP-MAX} \leq \lambda_{\min}^{MP}$	$P_{gi}^{\max}(\frac{\lambda_{\max}^{MP}+\lambda_{\min}^{MP}}{2} - a_i^*)$
$\lambda_{\min}^{MP} \leq a_i^* \leq \lambda_{\max}^{MP}$	$a_i^{MP-MAX} = a_i^*$	$\frac{P_{gi}^{\max}(\lambda_{\max}^{MP} - a_i^*)^2}{2(\lambda_{\max}^{MP} - \lambda_{\min}^{MP})}$
$a_i^* \geq \lambda_{\max}^{MP}$	$a_i^{MP-MAX} \geq \lambda_{\max}^{MP}$	0

TABLE III
EXPECTED PROFIT UNDER PAB WITH UNIFORMLY RANDOM λ^{PAB} SUCH THAT $\lambda_{\min}^{\mathrm{PAB}} \leq \lambda^{\mathrm{PAB}} \leq \lambda_{\max}^{\mathrm{PAB}}$

Cases	Condition	Expected Profit, $m_i^{MP}(a_i^{MP})$
A	$a_i^{PAB} \leq \lambda_{\min}^{PAB}$	$P_{gi}^{\max}(a_i^{PAB} - a_i^*)$
B	$\lambda_{\min}^{PAB} \leq a_i^{PAB} \leq \lambda_{\max}^{PAB}$	$\frac{P_{gi}^{\max}(a_i^{PAB} - a_i^*)}{\lambda_{\max}^{PAB} - \lambda_{\min}^{PAB}}(\lambda_{\max}^{PAB} - a_i^{PAB})$
C	$a_i^{PAB} \geq \lambda_{\max}^{PAB}$	0

TABLE IV
STRATEGIC OFFERS UNDER PAB WITH UNIFORMLY RANDOM λ^{PAB} SUCH THAT $\lambda_{\min}^{\mathrm{PAB}} \leq \lambda^{\mathrm{PAB}} \leq \lambda_{\max}^{\mathrm{PAB}}$

Condition	Strategic Offer	Maximum E{Profit}
$a_i^* \leq 2\lambda_{\min}^{PAB} - \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} = \lambda_{\min}^{PAB}$	$P_{gi}^{\max}(\lambda_{\min}^{PAB} - a_i^*)$
$2\lambda_{\min}^{PAB} - \lambda_{\max}^{PAB} \leq a_i^* \leq \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} = \frac{\lambda_{\max}^{PAB} + a_i^*}{2}$	$\frac{P_{gi}^{\max}(\lambda_{\max}^{PAB} - a_i^*)^2}{4(\lambda_{\max}^{PAB} - \lambda_{\min}^{PAB})}$
$a_i^* \geq \lambda_{\max}^{PAB}$	$a_i^{PAB-MAX} \geq \lambda_{\max}^{PAB}$	0

B. Strategic Offer Under PAB

Under PAB, the expected value of the profit of unit i is also found by solving (11), but this time over the possibly different range of SMC, $[\lambda_{\min}^{\mathrm{PAB}}, \lambda_{\max}^{\mathrm{PAB}}]$. Moreover, relation (8) is now used to define the profit function for an arbitrary offer parameter, a_i^{PAB}. The mean profit integral under PAB (11) can also be solved analytically and the results are summarized in Table III. The resulting expected profit as a function of the offer parameter a_i^{PAB} is denoted by $m_i^{\mathrm{PAB}}(a_i^{\mathrm{PAB}})$.

The probabilistic strategic offer of unit i under PAB that maximizes its expected profit is defined by

$$\max_{a_i^{\mathrm{PAB}}} \left\{ m_i^{\mathrm{PAB}}\left(a_i^{\mathrm{PAB}}\right)\right\}. \tag{14}$$

In Appendix A, we show that the strategic offer that solves (14), denoted by $a_i^{\mathrm{PAB-MAX}}$, and the corresponding maximum, denoted by $m_i^{\mathrm{PAB-MAX}} = m_i^{\mathrm{PAB}}(a_i^{\mathrm{PAB-MAX}})$, can be found analytically as shown in Table IV.

C. Comparing PAB and MP Strategic Offers

Referring to Table II, for MP we note that under two of the conditions on the true incremental cost, a_i^*, and the SMC uncertainty limits, the maximum expected profit can be obtained by nonunique strategic offers. However, under all conditions, the unique strategic offer $a_i^{\mathrm{MP-MAX}} = a_i^*$ always gives the maximum possible expected-profit. This result concurs with the well known idea that the best strategy under MP is to offer the true cost [5], [6], [8], even in the presence of SMC uncertainty.

From Table IV, we note that under PAB the probabilistic strategic offer differs significantly from the best strategy under MP. Here, as expected, the best offer is not at true cost, but a combination of the true cost and the uncertainty limits.

Other than the above comments, one cannot claim at this stage that one pricing strategy is superior or inferior to the other since the SMC uncertainty ranges under PAB and MP are different and have not yet been linked by common demand conditions. This is developed in the second part of this paper [10].

V. CONCLUSION

This paper develops generator strategic offers in perfect electricity markets operating under marginal and pay-as-bid pricing.

Recognizing that the system marginal cost cannot be predicted exactly, it is modeled as a random variable with known probability density function developed from historical data. The strategic generator offers under PAB and MP are then those that maximize the corresponding expected profit. These strategic offers are derived as analytic expressions assuming that the SMC is uniformly distributed over a known range which usually differs between the two pricing methods.

Among the possible strategic offers under MP, one of these is the true cost offer. Under PAB, however, the strategic offer is a combination of the true cost and the SMC uncertainty limits.

A fair and quantitative comparison of the two pricing methods under their respective strategic offers cannot be done at this stage, since such a comparison requires that the SMC uncertainty limits under PAB and MP correspond to a common set of loading conditions. This comparison is the subject of Part II of this paper.

APPENDIX I

A. Maximum Expected-Profit Under MP

The expected profit $m_i^{\mathrm{MP}}(a_i^{\mathrm{MP}})$ is a piece-wise function defined in Table I for three possible ranges of the offer parameter, a_i^{MP}. The maximum expected profit problem reduces then to maximizing $m_i^{\mathrm{MP}}(a_i^{\mathrm{MP}})$ over the offer parameter a_i^{MP} for the each of the three ranges and taking the one that yields the highest expected profit.

For any value of a_i^{MP} in the offer range defined by Case A, the expected profit is a constant given by $P_{gi}^{\max}((\lambda_{\max}^{\mathrm{MP}} + \lambda_{\min}^{\mathrm{MP}}/2) - a_i^*)$, however this expected profit can also be reached when $a_i^{\mathrm{MP}} = \lambda_{\min}^{\mathrm{MP}}$ in case B. Similarly, for the range of offers defined by case C, the expected profit is always zero, a value that can also be obtained in case B when $a_i^{\mathrm{MP}} = \lambda_{\max}^{\mathrm{PAB}}$. The conclusion then is that to find the maximum expected profit over cases

A,B, and C we need only calculate the maximum for Case B, in other words to maximize the quadratic function

$$m_i^{\mathrm{MP}}\left(a_i^{\mathrm{MP}}\right)=\frac{P_{gi}^{\max}}{\lambda_{\max}^{\mathrm{MP}}-\lambda_{\min}^{\mathrm{MP}}}\left[-\frac{\left(a_i^{\mathrm{MP}}\right)^2}{2}+a_i^* a_i^{\mathrm{MP}}\right.\left.+\frac{\left(\lambda_{\max}^{\mathrm{MP}}\right)^2}{2}-a_i^*\lambda_{\max}^{\mathrm{MP}}\right] \quad (15)$$

over the range, $\lambda_{\min}^{\mathrm{MP}} \leq a_i^{\mathrm{MP}} \leq \lambda_{\max}^{\mathrm{MP}}$. This constrained maximum occurs when

$$a_i^{\mathrm{MP-MAX}}=\begin{cases}\lambda_{\min}^{\mathrm{MP}}, & \text{if } a_i^* \leq \lambda_{\min}^{\mathrm{MP}}\\ a_i^*, & \text{if } \lambda_{\min}^{\mathrm{MP}} \leq a_i^* \leq \lambda_{\max}^{\mathrm{MP}}\\ \lambda_{\max}^{\mathrm{MP}}, & \text{if } a_i^* \geq \lambda_{\max}^{\mathrm{MP}}.\end{cases} \quad (16)$$

However, by comparing cases B and C, if $a_i^* > \lambda_{\max}^{\mathrm{MP}}$ then any offer a_i^{MP} satisfying $a_i^{\mathrm{MP}} \geq \lambda_{\max}^{\mathrm{MP}}$ will yield the same maximum expected profit of zero. Similarly, by comparing cases B and A, if $a_i^* < \lambda_{\min}^{\mathrm{MP}}$, then any offer satisfying $a_i^{\mathrm{MP}} \leq \lambda_{\min}^{\mathrm{MP}}$ will yield the same maximum expected profit of $P_{gi}^{\max}((\lambda_{\max}^{\mathrm{MP}}+\lambda_{\min}^{\mathrm{MP}}/2)-a_i^*)$. Combining these observations with (16), the MP strategic offer of unit i over all SMCs is

$$a_i^{\mathrm{MP-MAX}}\begin{cases}\leq \lambda_{\min}^{\mathrm{MP}}, & \text{if } a_i^* \leq \lambda_{\min}^{\mathrm{MP}}\\ = a_i^*, & \text{if } \lambda_{\min}^{\mathrm{MP}} \leq a_i^* \leq \lambda_{\max}^{\mathrm{MP}}\\ \geq \lambda_{\max}^{\mathrm{MP}}, & \text{if } a_i^* \geq \lambda_{\max}^{\mathrm{MP}}.\end{cases} \quad (17)$$

B. Maximum Expected-Profit Under PAB

Under PAB, the analytic expression for the expected profit, $m_i^{\mathrm{PAB}}(a_i^{\mathrm{PAB}})$, in terms of the offer parameter, a_i^{PAB}, is shown in Table III. This function is then maximized over all feasible a_i^{PAB} to obtain the best expected profit strategy. This maximization is subject to the a_i^{PAB} inequality defining the case and to the constraint $a_i^{\mathrm{PAB}} \geq a_i^*$ common to all three cases guaranteeing a nonnegative profit.

As with MP, this piece-wise optimization problem would have to be solved for each case, taking the one that yielded the highest of the three maxima. Here, we again argue that the highest maximum profit is always given by the maximization problem of Case B. This is so since the value of the objective function, $m_i^{\mathrm{PAB}}(a_i^{\mathrm{PAB}})$, is continuous at $a_i^{\mathrm{PAB}} = \lambda_{\min}^{\mathrm{PAB}}$ and $a_i^{\mathrm{PAB}} = \lambda_{\max}^{\mathrm{PAB}}$, and since the maximum expected profit in Case A at $a_i^{\mathrm{PAB}} = \lambda_{\min}^{\mathrm{PAB}}$ can also be reached in Case B with the same offer. Similarly, the best expected profit in Case C of zero can be obtained with the offer $a_i^{\mathrm{PAB}} = \lambda_{\max}^{\mathrm{PAB}}$, a maximum that can also be reached in Case B with the same offer.

For Case B, the expected profit is a quadratic function in a_i^{PAB}, that is,

$$m_i^{\mathrm{PAB}}\left(a_i^{\mathrm{PAB}}\right)=\frac{P_{gi}^{\max}}{\lambda_{\max}^{\mathrm{PAB}}-\lambda_{\min}^{\mathrm{PAB}}}\left[-\left(a_i^{\mathrm{PAB}}\right)^2\right.\left.+\left(\lambda_{\max}^{\mathrm{PAB}}+a_i^*\right)\right)a_i^{\mathrm{PAB}}-a_i^*\lambda_{\max}^{\mathrm{PAB}}\right] \quad (18)$$

whose maximum over the range $\lambda_{\min}^{\mathrm{PAB}} \leq a_i^{\mathrm{PAB}} \leq \lambda_{\max}^{\mathrm{PAB}}$ occurs when

$$a_i^{\mathrm{PAB-MAX}}=\begin{cases}\lambda_{\min}^{\mathrm{PAB}}, & \text{if } a_i^* \leq 2\lambda_{\min}^{\mathrm{PAB}}-\lambda_{\max}^{\mathrm{PAB}}\\ \frac{\left(\lambda_{\max}^{\mathrm{PAB}}+a_i^*\right)}{2}, & \text{if } 2\lambda_{\min}^{\mathrm{PAB}}-\lambda_{\max}^{\mathrm{PAB}} \leq a_i^* \leq \lambda_{\max}^{\mathrm{PAB}}\\ \lambda_{\max}^{\mathrm{PAB}}, & \text{if } a_i^* \geq \lambda_{\max}^{\mathrm{PAB}}.\end{cases} \quad (19)$$

However, we note that if $a_i^* > \lambda_{\max}^{\mathrm{PAB}}$, any offer such that $a_i^{\mathrm{PAB-MAX}} > \lambda_{\max}^{\mathrm{PAB}}$ yields the same profit of zero. Thus, the PAB strategic offer of unit i over all SMCs is

$$a_i^{\mathrm{PAB-MAX}}\begin{cases}= \lambda_{\min}^{\mathrm{PAB}}, & \text{if } a_i^* \leq 2\lambda_{\min}^{\mathrm{PAB}}-\lambda_{\max}^{\mathrm{PAB}}\\ = \frac{\left(\lambda_{\max}^{\mathrm{PAB}}+a_i^*\right)}{2} & \text{if } 2\lambda_{\min}^{\mathrm{PAB}}-\lambda_{\max}^{\mathrm{PAB}} \leq a_i^* \leq \lambda_{\max}^{\mathrm{PAB}}\\ \geq \lambda_{\max}^{\mathrm{PAB}} & \text{if } a_i^* \geq \lambda_{\max}^{\mathrm{PAB}}\end{cases}. \quad (20)$$

APPENDIX II

A more general model for the offered generation cost by generating unit i is to specify M_i linear segments or generation blocks, each characterized by its offered incremental cost and maximum output, $[a_i^k, P_{gi}^{\max k}; k = 1,\ldots,M]$. Strategically, the arbitrary generator i could submit an offer for each block, a_i^k, that differs from the true block incremental cost, a_i^{*k}. Then, the expected profit of generator i is as follows:

$$\begin{aligned} m_i &= \frac{1}{\lambda_{\max}-\lambda_{\min}}\int_{\lambda_{\min}}^{\lambda_{\max}}\sum_{k=1}^{M_i}\mathrm{pr}_i^k\left(\lambda, a_i^k\right)d\lambda \\ &= \frac{1}{\lambda_{\max}-\lambda_{\min}}\sum_{k=1}^{M_i}\int_{\lambda_{\min}}^{\lambda_{\max}} pr_i^k\left(\lambda, a_i^k\right)d\lambda \\ &= \sum_{k=1}^{M_i} m_i^k. \end{aligned} \quad (21)$$

Since the profit of block $k, pr^k(\lambda, a_i^k)$, is a function of the SMC, λ, of the offer, a_i^k, and of the true incremental cost a_i^{*k}, which is a known parameter, the mean profit m_i^k of segment k then is a function only over the variable a_i^k. Therefore, the problem of maximizing the expected profit of generator i can be decomposed into maximizing the expected profit for each block, that is

$$\max_{a_i^k;k=1\ldots M}\{m_i\}=\max_{a_i^1}\left\{m_i^1\right\}+\cdots+\max_{a_i^M}\left\{m_i^M\right\}. \quad (22)$$

In other words, by following the single-block strategic offer for each block, a generator with multi-block offers will maximize its expected profit.

If the true generation cost contains a fixed cost component, c_{0i}^*, plus M_i blocks, a possible offer strategy is for each block to independently use the single-block strategy, with the fixed cost added to the first block, thus increasing its break-even SMC value. However, under this method it is possible for the first block not to be scheduled while subsequent blocks are. This infeasible result can be avoided however by combining all

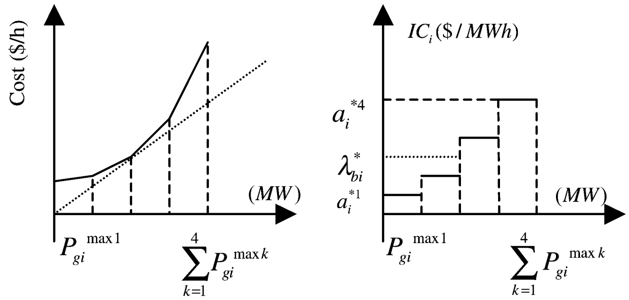

Fig. 1. Offered cost curve with four segments, or offered incremental cost curve with four blocks.

segments with incremental costs less than the break-even SMC into one block [5]. Only then, can the single-block strategy work with all remaining segments being selected sequentially according to their incremental cost order. This strategy is further clarified through the following example.

Fig. 1 shows the cost curve of generator i with four segments and its associated incremental costs. The break-even SMC, denoted by λ_{bi}^*, is defined by the condition that the SMC be equal to the average generation cost. In this example, $\lambda_{bi}^* = (c_{0i}^* + a_i^{*1} P_{gi}^{\max 1} + a_i^{*2} P_{gi}^{\max 2})/(P_{gi}^{\max 1} + P_{gi}^{\max 2})$, which is the slope of the dotted line in the left graph tangent to the cost curve. Then, the first two segments are combined into one, resulting in three incremental cost blocks now defined by $[\lambda_{bi}^*, P_{gi}^{\max 1} + P_{gi}^{\max 2}]$, $[a_i^{*3}, P_{gi}^{\max 3}]$, and $[a_i^{*4}, P_{gi}^{\max 4}]$. Since these segments are monotonically increasing, the problem is convex and the blocks will be scheduled sequentially. Then, the strategic offers derived for MP and PAB can be applied to each block as if it were an independent generating unit.

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