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Resolving the deposit dilemma: A new DEA bank efficiency model

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ABSTRACT

One of the weaknesses of current bank efficiency models is a disagreement as to the role of deposits in the bank production process. Some models view deposits as an input, while others view them as an output. Such disparity of approaches results in inconsistent efficiency estimates. In this study we propose an alternative Data Envelopment Analysis (DEA) bank efficiency model that treats deposits as an intermediate product, thus emphasizing the dual role of deposits in the bank production process. Consequently, the effect of the amount of deposits on bank efficiency depends on the efficiency at both stages of the bank production process. The main advantage of our model is that it does not require a researcher to make a judgment call as to whether having more (production approach) or less (intermediation approach) deposits is "better" for bank efficiency. Our unified framework has the potential to produce more consistent efficiency estimates.

1. Introduction

Evaluating the efficiency in the banking industry has been a focus of numerous research studies since the early works by Greenbaum (1967), Benston (1965) and others. Nevertheless, there is still a controversy in the literature as to the "right" way to estimate bank efficiency. The main confusion stems from the disagreement among the researchers about the appropriate inputs and outputs in the production process of a banking firm. It is widely accepted that fixed assets and employees serve as main inputs, while different kinds of earning assets represent outputs of a bank. However, the role of bank liabilities, particularly deposits, is quite controversial. The two main approaches to treating bank deposits are the production approach and the financial intermediation approach. Under the production approach, deposits are treated as outputs, because they are viewed as a service provided by a bank to its customers. On the other hand, the financial intermediation approach views banks as intermediaries that take deposits and make loans. Consequently, deposits are considered to be an input for the production of loans and other earning assets.

Facing a dilemma of whether to treat deposits as an input or an output, the researchers typically take either the production or the financial intermediation approach with no overwhelming preference to one or the other. This is not surprising given that each approach has its merits and can be theoretically justified. Unfortunately, such a divide in the literature creates inconsistency in the efficiency estimates across the studies. A bank that has relatively more deposits and fewer loans will be considered inefficient under the financial intermediation approach, but may be considered efficient under the production approach. As a result, the decision whether to consider deposits as an input or an output may have a major impact on the obtained efficiency measures.

In this study we propose a novel Data *Envelopment Analysis* (DEA) model of bank efficiency that treats deposits as neither an input nor an output. Instead, we consider deposits to be an intermediate product that is an output from the first stage of the bank production process and is an input to the second stage. As a result, we impose no judgment on whether larger or smaller values of deposits are more desirable. The effect of deposits on bank efficiency is non-trivial and is determined by the combined efficiency scores at both stages of the production process. When building an efficient reference bank for a particular bank under consideration, the reference bank will consume no more of each input (employees and fixed assets) and produce at least as much of each output (loans and other earning assets). However, we force the reference bank to have the same amount of deposits as the bank under consideration. In other words, we ask the following question: given a certain amount of deposits, to what levels can a bank reduce its inputs and increase its outputs? There are two advantages of our model over the previous models developed in the literature. First,

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we believe that modeling deposits as an intermediate product reflects the actual bank production process more realistically. Second, we eliminate the need to decide whether having more deposits (production approach) or fewer deposits (intermediation approach) is "better" for bank efficiency, thus proposing a framework that has the potential to produce more consistent efficiency estimates across the studies.

We provide background to place our study in the context of the existing literature in Section 2. In Section 3, we provide a brief discussion of DEA and *network DEA* which extends the basic DEA methodology to processes with more complex internal structure. In Section 4, we present the model formulation. In Section 5, we apply our model to obtain estimates of the efficiency scores for a sample of bank holding companies. Finally, Section 6 contains our conclusions.

2. Background

Estimating efficiency in the financial industry involves identifying the efficient frontier as a benchmark for measuring relative performance of the units. The relative efficiency score of a banking organization is determined by how close it is to the efficient frontier. The methods of identifying the efficient frontier can be grouped in two broad categories: nonparametric and parametric. Nonparametric methods that include Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH) do not put any restrictions on the functional form of the relationship between inputs and outputs. This feature of nonparametric methods is particularly appealing for estimating efficiency of financial institutions, which do not have a well defined production function. Parametric methods, such as the Stochastic Frontier Approach (SFA), the Distribution-Free Approach (DFA), and the Thick Frontier Approach (TFA), assume a specific functional form for the cost, profit, or production function. This restrictive nature of the parametric methods is their main disadvantage compared to the nonparametric methods. On the other hand, parametric methods allow for a random error in the estimation process, while nonparametric methods do not. There is no agreement in the literature as to which of the methods is preferable. Both approaches have been widely used in the literature. Berger and Humphrey (1997) provide an excellent survey of studies on financial efficiency, where they discuss different methodologies and track their applications across the studies.

Regardless of the method utilized for the efficiency estimation, it is crucial to identify the appropriate inputs and outputs of a banking organization. Unfortunately, due to the nature of the operation of a banking firm, the definition of its inputs and outputs is controversial. Most researchers agree that employees and fixed assets such as buildings and equipment represent bank inputs. On the other hand, loans and other earning assets are mostly viewed as bank outputs. The role of deposits and other types of liabilities, however, is controversial. Some researchers believe that deposits should be considered as bank outputs, since they represent the service provided to the customers (production approach). Others emphasize the role of banks as financial intermediaries that use deposits and other liabilities in order to make loans and invest in other earning assets. According to this intermediation approach, deposits should be treated as inputs of a bank. Sealey and Lindley (1977) provide a theoretical discussion of both approaches.

The dual role of deposits in the bank production process forces the researchers to take either the production or financial intermediation approach, and, therefore, make a rather arbitrary choice of bank inputs and outputs when estimating bank efficiency. Numerous empirical studies, including Aly et al. (1990), Zaim (1995), DeYoung and Nolle (1998), Berger and Mester (1997), DeYoung and Hasan (1998), Isik and Hassan (2002), Beccalli et al. (2006), Lozano-Vivas and Pasiouras (2010), Banker et al. (2010), Hsiao et al. (2010) adopt the intermediation approach. On the other hand, Berger et al. (1987), Hunter and Timme (1995), Berger and DeYoung (1997), Resti (1997), Devaney and Weber (2002), Glass et al. (2010), among others, use the production approach. The decision to follow one of the above mentioned approaches is usually driven by personal preferences of the authors. Unfortunately, whether deposits enter a model as an input or an output may have a significant effect on the efficiency results obtained from the model estimation. Everything else being equal, a bank that has relatively more deposits compared to other banks will be considered relatively efficient under the production approach and will be deemed relatively inefficient under the intermediation approach. For example, Hunter and Timme (1995) estimate various DFA specifications and find the efficiency estimates to be rather sensitive to whether the deposits are considered as an input or an output. In particular, they find statistically significant differences in the mean efficiency scores produced by the various specifications. Moreover, they show that the rankings of the individual banks are weakly correlated across specifications.

The sensitivity of the efficiency scores to the specification of inputs and outputs undermines the ability of the above mentioned methodologies to be applied to individual bank performance evaluation. Indeed, the consistency of absolute and relative bank efficiency estimates is vital in evaluating bank mergers, market structure, control issues, and other applications that require across-bank comparisons. Therefore, it is crucial to develop a methodology that would rid the researchers of making a judgment call about the appropriate inputs and outputs, thus providing a framework for more consistent estimation of bank efficiency.

In this study we try to fill the apparent gap in the literature by developing a DEA model that attempts to resolve the "deposit dilemma." Instead of treating deposits as an input or an output, we consider deposits to be an intermediate product that is an output from the first stage of the bank production process and is an input to the second stage. We ask to what levels can a bank reduce its "true" inputs (employees and fixed assets) and increase its "true" outputs (loans and other earning assets), *given* a certain amount of deposits. In our model there is no need to assume whether having a higher or lower dollar value of deposits is "better" for bank efficiency. Instead, the effect of deposits on the overall bank efficiency is determined by the bank's relative efficiency at each stage of production. As a result, we have a model that recognizes the importance of deposits in the production process of a bank, and, at the same time, avoids the confusion associated with considering deposits as an input or an output.

3. Data Envelopment Analysis and network DEA

Data Envelopment Analysis has become a widely used methodology for evaluating *relative efficiency*. We trace its mathematical development to Charnes et al. (1978), who built on the work of Farrell (1957) and others. DEA measures relative efficiency in situations in which there are multiple inputs and outputs and there is no obvious objective way to aggregate either inputs or outputs into a meaningful index of productive efficiency. The technique is well documented in the management science literature (Charnes et al., 1978, 1981; Sexton, 1986; Cooper et al., 1999), and it has received increasing attention as researchers have wrestled with problems of productivity measurement, especially in the services and nonmarket sectors of the economy. As was discussed in the previous section, DEA is also one of the most common methodologies applied to the efficiency estimation in the banking industry.

In its basic form, DEA considers a collection of *decision-making units* (DMUs) each of which consumes DMU-specific levels of selected inputs to produce DMU-specific levels of selected outputs. DEA makes no assumptions regarding the manner in which a DMU converts inputs into outputs; each DMU is a "black box" with respect to its production process. DEA models allow for differing assumptions regarding *returns-to-scale*. In addition, DEA models may be *input-oriented*, *output-oriented*, or *unoriented*. Input-oriented models identify input reductions that would enable a DMU to become efficient while output-oriented models identify output increases that would achieve the same effect. Unoriented models identify a mix of input reductions and output increases that lead to efficiency.

DEA establishes an *efficient frontier* based on observed best performances and evaluates the efficiency of each DMU relative to this frontier. DMUs that lie on the frontier are *efficient*. DEA evaluates the efficiency of a DMU that does not lie on the frontier relative to a linear combination of the efficient DMUs. This linear combination represents an empirically feasible *reference DMU* that dominates the inefficient DMU under evaluation. The reference DMU consumes no more of each input while producing at least as much of each output as does the DMU under evaluation. The DEA model finds the most productive reference DMU and computes the efficiency of the DMU under evaluation relative to this reference DMU. For example, if the reference DMU produces at least 25% more of every output while consuming no more of each input, then the *inverse efficiency* of the DMU under evaluation is 1.25 and its efficiency is 1/1.25 = 0.8. We can formulate the DEA model for a specific DMU as a mathematical program. A complete DEA requires that we solve one such mathematical program for each DMU.

As stated above, DEA models treat the DMU as a "black box." Inputs enter and outputs exit, with no consideration of the underlying process. Consequently, it is difficult to provide individual DMU managers with specific information regarding the sources of inefficiency within their DMUs. Network DEA allows the analyst to look inside the DMU, allowing greater insight as to the sources of organizational inefficiency. In network DEA, each DMU is comprised of two or more *sub-DMUs*. Each resource consumed by a sub-DMU either enters the DMU from outside (input to the DMU) or is produced by another sub-DMU (intermediate product). Each product produced by a sub-DMU either exits the DMU (output of the DMU) or is consumed by another sub-DMU (intermediate product). A typical network DEA DMU is presented in Fig. 1.

Many researchers (Färe and Whittaker, 1995; Färe and Grosskopf, 2000; Castelli et al., 2001; Sexton and Lewis, 2003; Lewis and Sexton, 2004; Lewis et al., 2010) have proposed various approaches to network DEA. Network DEA models can be input-oriented, output-oriented, or unoriented. In addition, we can incorporate differing assumptions regarding returns-to-scale in network DEA models.

We focus on the unoriented network DEA methodology presented in Lewis et al. (2010), to emphasize the importance of simultaneously decreasing inputs and increasing outputs in a banking organization. To determine the efficiency and inverse efficiency at each sub-DMU, we solve a standard unoriented DEA model using the actual levels of inputs (or intermediate products) consumed and outputs (or intermediate products) produced by that sub-DMU. Then, to evaluate the organizational efficiency and inverse efficiency, we apply an iterative process which alternates between each sub-DMU. At a particular iteration, we incorporate hypothetical target levels of the inputs and outputs from the previous iteration on the RHS of the relevant constraints. In addition, we incorporate hypothetical target levels of intermediate products from the alternate stage on the RHS of the relevant constraints. We continue until the efficiency scores of all sub-DMUs during an iteration equal 1. Finally, we determine the organizational efficiency and inverse efficiency from the ratios of the final hypothetical levels of inputs and outputs to their actual levels, respectively.

To demonstrate the unoriented network DEA methodology, we consider a *two-stage DEA* model consisting of one input, x_k, one intermediate product, y_k, and one output, z_k as shown in Fig. 2. A two-stage DEA model is a special case of network DEA in which there are exactly two sub-DMUs connected in series. For the given level of input, x_k and intermediate product, y_k the stage 1 sub-DMU could consume $x_k^* \leqslant x_k$ units of input and produce $y_{1k}^* \geqslant y_k$ units of intermediate product. Thus, the efficiency of the stage 1 sub-DMU is $\varepsilon_{1k} = \frac{x_k^*}{x_k} \leqslant 1$ and its inverse efficiency is $\theta_{1k} = \frac{y_{1k}^*}{y_k} \geqslant 1$. For the given level of intermediate product, y_k and output z_k the stage 2 sub-DMU could consume $y_{2k}^* \leqslant y_k$ units of intermediate product and produce $z_k^* \geqslant z_k$ units of output. Thus, the efficiency of the stage 2 sub-DMU is $\varepsilon_{2k} = \frac{y_{2k}^*}{y_k} \leqslant 1$ and its inverse efficiency is $\theta_{2k} = \frac{z_k^*}{z_k} \geqslant 1$.

Next, to determine the organizational efficiency and inverse efficiency, we apply the iterative process. For the levels of input,

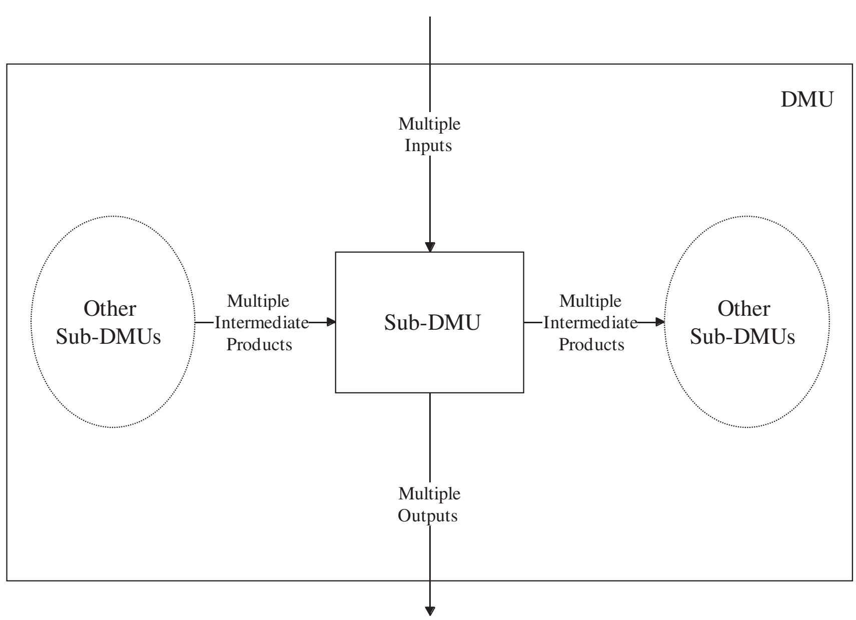

Fig. 1. Structure of a typical sub-DMU in a network DEA model.

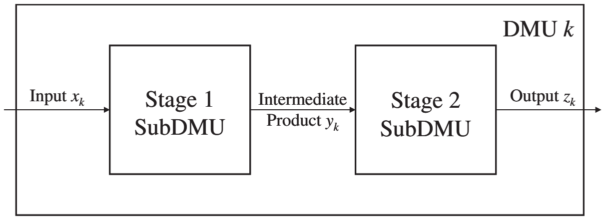

Fig. 2. Two-stage DEA model with one input, one intermediate product, and one output.

x_k and intermediate product, y_k, the stage 1 sub-DMU could consume $x^*_{k1} \leqslant x_k$ units of input and produce $y^*_{1k1} \geqslant y_k$ units of intermediate product. For the levels of intermediate product, y^*_{1k1} and output, z_k, the stage 2 sub-DMU could consume $y^*_{2k1} \leqslant y^*_{1k1}$ units of intermediate product and produce $z^*_{k1} \geqslant z_k$ units of output. This completes iteration 1. For the levels of input, x^*_{k1} and intermediate product, y^*_{2k1}, the stage 1 sub-DMU could consume $x^*_{k2} \leqslant x^*_{k1}$ units of input and produce $y^*_{1k2} \geqslant y^*_{2k1}$ units of intermediate product. For the levels of intermediate product, y^*_{1k2} and output, z^*_{k1}, the stage 2 sub-DMU could consume $y^*_{2k2} \leqslant y^*_{1k2}$ units of intermediate product and produce $z^*_{k2} \geqslant z^*_{k1}$ units of output. This completes iteration 2. In general, for the levels of input, x^*_{kt-1} and intermediate product, y^*_{2kt-1}, the stage 1 sub-DMU could consume $x^*_{kt} \leqslant x^*_{kt-1}$ units of input and produce $y^*_{1kt} \geqslant y^*_{2kt-1}$ units of intermediate product. For the levels of intermediate product, y^*_{1kt} and output, z^*_{kt-1}, the stage 2 sub-DMU could consume $y^*_{2kt} \leqslant y^*_{1kt}$ units of intermediate product and produce $z^*_{kt} \geqslant z^*_{kt-1}$ units of output. Table 1 and Fig. 3 summarize the iterative process. As shown in Figs. 4–6, the input level, intermediate product level, and output level eventually converge to some values x^{**}_k, y^{**}_k, and z^{**}_k, respectively. The organizational efficiency and inverse efficiency can be determined from the ratio of the final input level to the initial input level $\varepsilon_k = \frac{x^{**}_k}{x_k}$ and the ratio of the final output level to the initial output level $\theta_k = \frac{z^{**}_k}{z_k}$, respectively.

4. The model

Fig. 7 shows our proposed model of a bank's production process. Let FA_{j0} be the dollar amount of fixed assets at bank j, E_{j0} be the actual number of employees at bank j, D_{j0} be the dollar amount of deposits at bank j, L_{j0} be the dollar amount of loans at bank j, and EA_{j0} be the dollar amount of other earning assets at bank j. Define α_j and β_j to be the fraction of fixed assets and the fraction of employees, respectively, consumed at stage 1 of bank j.

As indicated in Fig. 7, we treat deposits as neither an input nor an output. Instead, we consider deposits to be an intermediate product linking the two stages. Indeed, banks use their fixed assets and employees to acquire deposits, make loans and invest in other earning assets. The deposits acquired in the first stage, however,

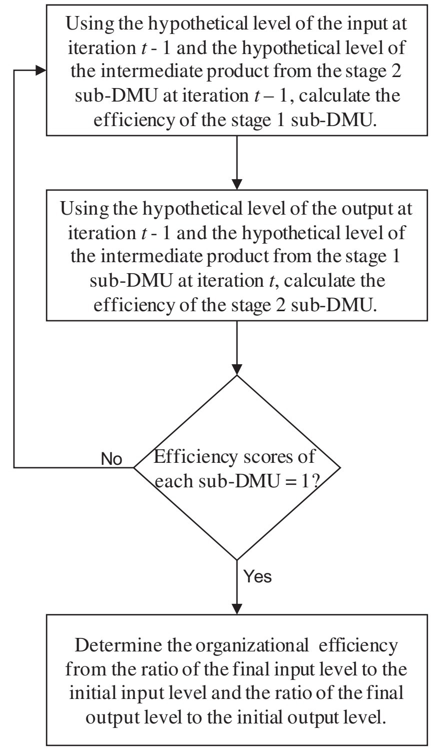

Fig. 3. Flowchart of the iterative process for determining the organizational efficiency and inverse efficiency of the two-stage DEA example assuming an unoriented model.

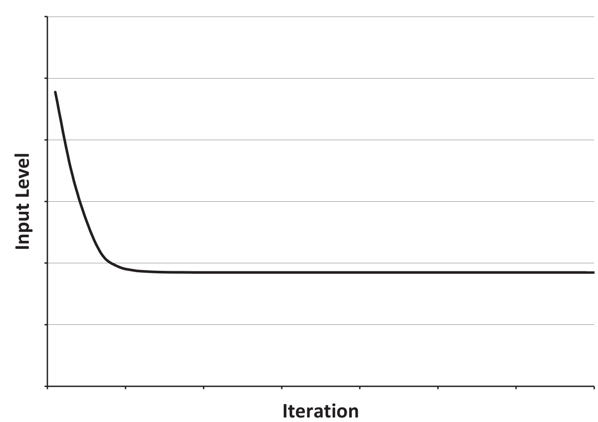

Fig. 4. Convergence of the input.

serve as the main source of funding for a bank's lending activity. Unlike the models that treat deposits as a pure input or a pure out-

Table 1
Summary of the iterative process for determining the organizational efficiency and inverse efficiency of the two-stage DEA example assuming an unoriented model.

Iteration	Stage	Input		Intermediate product		Output	
		Initial	Final	Initial	Final	Initial	Final
1	1	x_k	$x^*_{k1} \leqslant x_k$	y_k	$y^*_{lk1} \geqslant x_k$		
	2			y^*_{lk1}	$y^*_{2k1} \geqslant y^*_{1k1}$	z_k	$z^*_{k1} \geqslant z_k$
2	1	x^*_{k1}	$x^*_{k2} \leqslant x^*_{k1}$	y^*_{2k1}	$y^*_{1k2} \geqslant y^*_{2k1}$		
	2			y^*_{1k2}	$y^*_{2k2} \geqslant y^*_{1k2}$	z^*_{k1}	$z^*_{k2} \geqslant z^*_{k1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
t	1	x^*_{kt-1}	$x^*_{kt} \leqslant x^*_{kt-1}$	y^*_{2kt-1}	$y^*_{lkt} \geqslant y^*_{2kt-1}$		
	2			y^*_{lk1}	$y^*_{2kt} \leqslant y^*_{lkt}$	z^*_{kt-1}	$z^*_{k1}z^*_{kt-1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

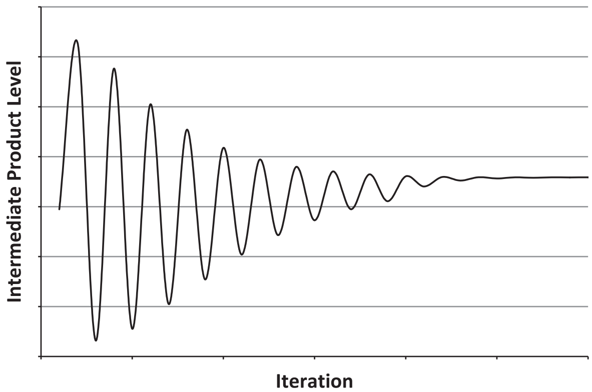

Fig. 5. Convergence of the intermediate product.

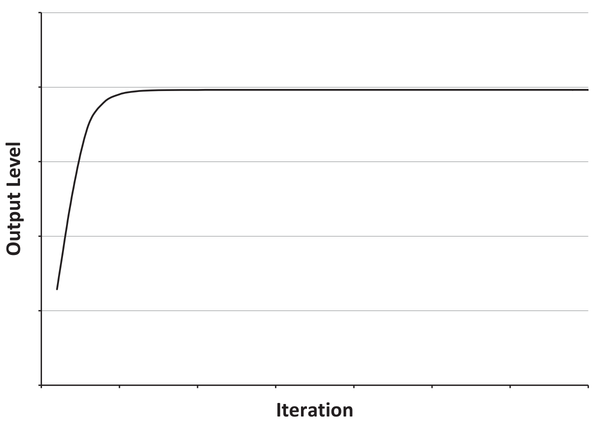

Fig. 6. Convergence of the output.

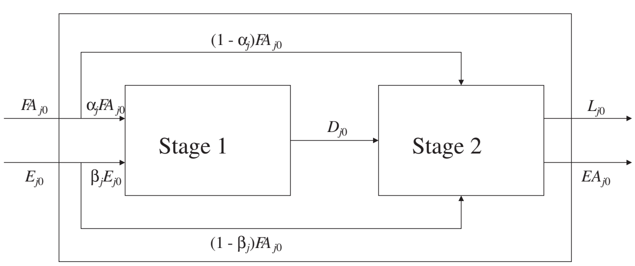

Fig. 7. Network DEA model of a bank.

put, our model emphasizes the dual role of deposits in the bank production process. Consequently, the effect of the amount of deposits on bank efficiency is non-trivial, and depends on the efficiency at both stages.

We can evaluate the efficiency of a bank using an unoriented variable returns to scale network DEA model. Our model is unoriented since we believe that bank managers seek to simultaneously decrease input levels and increase output levels. In addition, we assume variable returns to scale since we believe it to be unfair to compare "large" banks to "small" banks and vice versa. Our methodology follows from Lewis et al. (2010), which expands on the methodology proposed in Sexton and Lewis (2003) and Lewis and Sexton (2004) to allow for an unoriented model.

Let ε_{1kt} and ε_{2kt} to be the efficiencies of the stage 1 model and the stage 2 model for bank k during iteration t, respectively, and define θ_{1kt} and θ_{2kt} to be the approximate inverse efficiencies of the stage 1 model and the stage 2 model for bank k during iteration t, respectively. Define λ_{jt} to be the weight placed on bank j by bank k when solving the stage 1 model during iteration t and define μ_{jt} to be the weight placed on bank j by bank k when solving the stage 2 model during iteration t. Let λ^*_{jt} and μ^*_{jt} denote the optimal weights. The stage 1 model for bank k during iteration t can be formulated as follows:

$$\begin{aligned}
&\text{Min}\,\varepsilon_{1kt} \text{ or } \text{Max}\,\theta_{1kt}\\
&\text{subject to}\\
&\sum_{j=1}^{n}\lambda_{jt}\alpha_j FA_{j0} \leqslant \begin{cases}\varepsilon_{1kt}\alpha_k FA_{k0} & t=1\\ \varepsilon_{1kt}\sum_{j=1}^{n}\lambda^*_{jt-1}\alpha_j FA_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\lambda_{jt}\beta_j E_{j0} \leqslant \begin{cases}\varepsilon_{1kt}\beta_k E_{k0} & t=1\\ \varepsilon_{1kt}\sum_{j=1}^{n}\lambda^*_{jt-1}\beta_j E_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\lambda_{jt} D_{j0} \geqslant \begin{cases}\theta_{1kt} D_{k0} & t=1\\ \theta_{1kt}\sum_{j=1}^{n}\mu^*_{jt-1} D_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\lambda_{jt} = 1\\
&\varepsilon_{1kt}+\theta_{1kt}=2\\
&\lambda_{jt}\geqslant 0 \quad j=1,2,\ldots,n\\
&0\leqslant\varepsilon_{1kt}\leqslant 1\\
&\theta_{1kt}\geqslant 1
\end{aligned}$$

The objective function minimizes the stage 1 relative efficiency at bank k during iteration t (or equivalently maximizes its approximate stage 1 inverse efficiency). The first two constraints ensure that the hypothetical target bank for bank k at iteration t consumes no more of each stage 1 input (fixed assets and employees) than it does at iteration $t-1$. The third constraint ensures that the hypothetical target bank for bank k at iteration t generates at least as much of the intermediate product (deposits) as is consumed by stage 2 at iteration $t-1$.

The stage 2 model for bank k during iteration t can be formulated as follows:

$$\begin{aligned}
&\text{Min}\,\varepsilon_{2kt} \text{ or } \text{Max}\,\theta_{2kt}\\
&\text{subject to}\\
&\sum_{j=1}^{n}\mu_{jt}(1-\alpha_j)FA_{j0} \leqslant \begin{cases}\varepsilon_{2kt}(1-\alpha_k)FA_{k0} & t=1\\ \varepsilon_{2kt}\sum_{j=1}^{n}\mu^*_{jt-1}(1-\alpha_j)FA_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\mu_{jt}(1-\beta_j)E_{j0} \leqslant \begin{cases}\varepsilon_{2kt}(1-\beta_k)E_{k0} & t=1\\ \varepsilon_{2kt}\sum_{j=1}^{n}\mu^*_{jt-1}(1-\beta_j)E_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\mu_{jt}D_{j0} \leqslant \varepsilon_{2kt}\sum_{j=1}^{n}\lambda^*_{jt}D_{j0}\\
&\sum_{j=1}^{n}\mu_{jt}L_{j0} \geqslant \begin{cases}\theta_{2kt}L_{k0} & t=1\\ \theta_{2kt}\sum_{j=1}^{n}\mu^*_{jt-1}L_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\mu_{jt}EA_{j0} \geqslant \begin{cases}\theta_{2kt}EA_{k0} & t=1\\ \theta_{2kt}\sum_{j=1}^{n}\mu^*_{jt-1}EA_{j0} & t\geqslant 2\end{cases}\\
&\sum_{j=1}^{n}\mu_{jt} = 1\\
&\varepsilon_{2kt}+\theta_{2kt}=2\\
&\mu_{jt}\geqslant 0 \qquad j=1,2,\ldots,n\\
&0\leqslant\varepsilon_{2kt}\leqslant 1\\
&\theta_{2kt}\geqslant 1
\end{aligned}$$

The objective function minimizes the stage 2 relative efficiency at bank k during iteration t (or equivalently maximizes its approximate stage 2 inverse efficiency). The first two constraints ensure that the hypothetical target bank for bank k at iteration t consumes no more of each stage 2 input (fixed assets and employees) than that at iteration $t-1$. The third constraint ensures that the hypothetical target bank for bank k at iteration t consumes no more of the intermediate product (deposits) than is generated by stage 1 at iteration t. The fourth and fifth constraints ensure that the hypothetical target bank for bank k at iteration t generates at least as much of each stage 2 output (loans and other earning assets) as that at iteration $t-1$.

Note that the formulations presented above assume that we solve the stage 1 model first during each iteration. If we solve the stage 2 model first, then the third constraint in the stage 1 formulation becomes:

$$\sum_{j=1}^{n} \lambda_{jt} D_{j0} \geqslant \theta_{1kt} \sum_{j=1}^{n} \mu_{jt}^{*} D_{j0}$$

Finally, the third constraint in the stage 2 formulation becomes:

$$\sum_{j=1}^{n} \mu_{jt} D_{j0} \leqslant \begin{cases} \varepsilon_{2kt} D_{k0} & t = 1 \\ \varepsilon_{2kt} \sum_{j=1}^{n} \lambda_{jt-1}^{*} D_{j0} & t \geqslant 2 \end{cases}$$

Unfortunately, data limitations prevent the disaggregation of fixed assets and employees required by our model. That is, we do not know the values α_j and β_j for bank j. Thus, we propose a modified model that still puts no value judgment on deposits. Fig. 8 shows our modified model of a bank. A consequence of this modification is that, while we can evaluate the efficiency of the bank, we can no longer evaluate the efficiency of each stage. The advantage of the modified model is that we can continue to treat deposits as an intermediate product rather than as an input or an output.

We can simplify the notation used in our modified model. Let FA_j be the actual amount of fixed assets at bank j, E_j be the actual number of employees at bank j, D_j be the actual amount of deposits at bank j, L_j be the actual amount of loans at bank j, and EA_j be the actual amount of other earning assets at bank j. Define λ_j to be the weight placed on bank j by bank k, ε_k to be the relative efficiency of bank k, and θ_k to be the approximate inverse efficiency of bank k. The DEA model for bank k can be formulated as follows:

$$\begin{aligned}
&\text{Min}\, \varepsilon_k \text{ or } \text{Max}\, \theta_k \\
&\text{subject to} \\
&\quad \sum_{j=1}^{n} \lambda_j FA_j \leqslant \varepsilon_k FA_k \\
&\quad \sum_{j=1}^{n} \lambda_j E_j \leqslant \varepsilon_k E_k \\
&\quad \sum_{j=1}^{n} \lambda_j D_j = D_k \\
&\quad \sum_{j=1}^{n} \lambda_j L_j \geqslant \theta_k L_k \\
&\quad \sum_{j=1}^{n} \lambda_j EA_j \geqslant \theta_k EA_k \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad \varepsilon_k + \theta_k = 2 \\
&\quad \lambda_j \geqslant 0 \quad j = 1, 2, \ldots, n \\
&\quad 0 \leqslant \varepsilon_k \leqslant 1 \\
&\quad \theta_k \geqslant 1
\end{aligned}$$

The objective function minimizes the relative efficiency of bank k (or equivalently maximizes its approximate inverse efficiency). The first two constraints ensure that the hypothetical target bank consumes no more of each input (fixed assets and employees) than does bank k. The third constraint ensures that the hypothetical target bank has the same amount of deposits as does bank k. The fourth and fifth constraints ensure that the hypothetical target bank generates at least as much of each output (loans and other earning assets) as does bank k.

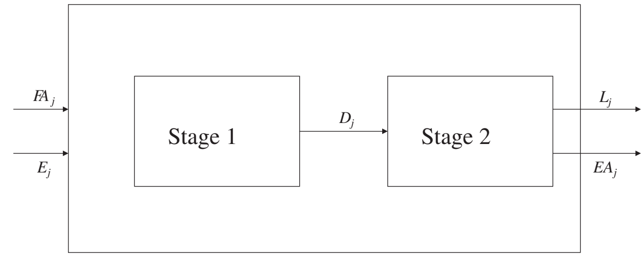

Fig. 8. Modified model of a bank.

5. Model estimation

We now demonstrate the working of our model by estimating the relative efficiency scores of banking organizations, using Consolidated Financial Statements for Bank Holding Companies (FR-Y9 reports). We collect December balance sheet data for the period of 1986–2008, and estimate our model for each year in that period. We restrict the sample to the top tier bank holding companies that file consolidated financial statements. We also require that for each year, each bank holding company has non-missing values for each input and output. Since the primary focus of our study is methodological, we do not aim at explaining the variation in efficiency across time and across banking organizations in our empirical analysis. Instead, we apply our model to the data and compare the results with those produced by the alternative models that use deposits as either an input or an output.

Our inputs are fixed assets and the number of employees. Our outputs are total loans and other earning assets, defined as the sum of securities, federal funds sold, and trading assets. As was described in the previous sections, deposits are treated as neither an input nor an output. Instead, the total deposits enter our model as an intermediate product. We also estimate two common alternative models. The first such model treats deposits as an input, while the second considers deposits to be an output.

Tables 2 and 3 present the mean, median, and other descriptive statistics of the efficiency scores for bank holding companies in each year of the study when treating deposits as an input and an output, respectively. For each year in the study, we performed a paired T-test and a Wilcoxon Signed Rank test to compare the efficiency scores when treating deposits as an input and as an output. Results indicate statistical significance ($P < 0.00005$ for all tests) between the two models. Thus, the choice of whether to treat deposits as an input or an output appears to affect the efficiency scores of bank holding companies.

Table 4 presents the mean, median, and other descriptive statistics of the efficiency scores for bank holding companies in each year of the study when treating deposits as an intermediate product. For each year in the study, we performed paired T-tests and Wilcoxon Signed Rank tests to compare our model to the models that treat deposits as an input and an output. In all but four instances the P-values were less than 0.00005. For 2005 and 1988, the Wilcoxon Signed Rank test returned a P-value of 0.0001 and 0.0014, respectively, when comparing our model to the model that treats deposits as an output. For 1987, the Paired T-test and the Wilcoxon Signed Rank test returned P-values of 0.1187 and 0.3357, respectively, when comparing our model to the model that

Table 2
Descriptive statistics of the efficiency scores obtained from a DEA model that treats deposits as an input. The sample includes top tier bank holding companies filing consolidated financial statements. Inputs include fixed assets and the number of employees. Outputs include total loans and other earning assets, defined as the sum of securities, federal funds sold, and trading assets. Total deposits are treated as an input.

Year	*N*	Mean	SD	Minimum	1st quartile	Median	3rd quartile	Maximum
1986	1280	0.7787	0.0791	0.5438	0.7228	0.7726	0.8233	1
1987	1340	0.7139	0.1021	0.5071	0.6372	0.6938	0.7748	1
1988	1366	0.7197	0.0988	0.3679	0.6537	0.7033	0.7668	1
1989	1372	0.7408	0.1027	0.4539	0.6739	0.7295	0.7977	1
1990	1433	0.6824	0.1320	0.1877	0.5959	0.6693	0.7640	1
1991	1447	0.7486	0.1085	0.2828	0.6790	0.7462	0.8147	1
1992	1464	0.7058	0.1404	0.1907	0.6214	0.7123	0.8021	1
1993	1454	0.7547	0.1038	0.3779	0.6864	0.7492	0.8140	1
1994	1190	0.7231	0.1079	0.4289	0.6526	0.7123	0.7755	1
1995	1214	0.7214	0.1100	0.3627	0.6524	0.7097	0.7772	1
1996	1270	0.6759	0.1263	0.3020	0.5937	0.6558	0.7417	1
1997	1373	0.7081	0.1059	0.3457	0.6417	0.6968	0.7622	1
1998	1473	0.6694	0.1086	0.2905	0.6014	0.6545	0.7192	1
1999	1523	0.6305	0.1192	0.2520	0.5502	0.6119	0.6867	1
2000	1625	0.6469	0.1236	0.2954	0.5627	0.6237	0.7137	1
2001	1729	0.6303	0.1215	0.2203	0.5507	0.6089	0.6876	1
2002	1860	0.6425	0.1137	0.2195	0.5662	0.6242	0.7001	1
2003	2036	0.6596	0.1158	0.2336	0.5816	0.6470	0.7242	1
2004	2147	0.6547	0.1181	0.1862	0.5751	0.6418	0.7182	1
2005	2158	0.6087	0.1081	0.1736	0.5351	0.5910	0.6608	1
2006	920	0.6254	0.1273	0.1700	0.5426	0.6024	0.6881	1
2007	899	0.6236	0.1302	0.2439	0.5385	0.5965	0.6894	1
2008	901	0.7538	0.1095	0.2263	0.6817	0.7464	0.8160	1

Table 3
Descriptive statistics of the efficiency scores obtained from a DEA model that treats deposits as an output. The sample includes top tier bank holding companies filing consolidated financial statements. Inputs include fixed assets and the number of employees. Outputs include total loans and other earning assets, defined as the sum of securities, federal funds sold, and trading assets. Total deposits are treated as an output.

Year	*N*	Mean	SD	Minimum	1st quartile	Median	3rd quartile	Maximum
1986	1280	0.5437	0.1581	0.0963	0.4328	0.5173	0.6373	1
1987	1340	0.5529	0.1536	0.1654	0.4431	0.5197	0.6392	1
1988	1366	0.5402	0.1681	0.1046	0.4181	0.5087	0.6406	1
1989	1372	0.5649	0.1638	0.0995	0.4452	0.5296	0.6535	1
1990	1433	0.5009	0.1790	0.1485	0.3665	0.4601	0.6013	1
1991	1447	0.5625	0.1475	0.1462	0.4640	0.5308	0.6383	1
1992	1464	0.5410	0.1520	0.1315	0.4388	0.5072	0.6075	1
1993	1454	0.5475	0.1542	0.1173	0.4460	0.5123	0.6119	1
1994	1190	0.5282	0.1573	0.1125	0.4244	0.4907	0.5872	1
1995	1214	0.5177	0.1598	0.0946	0.4155	0.4730	0.5771	1
1996	1270	0.4744	0.1684	0.0831	0.3662	0.4305	0.5265	1
1997	1373	0.4397	0.1640	0.0354	0.3367	0.3985	0.4930	1
1998	1473	0.3930	0.1699	0.0329	0.2881	0.3424	0.4327	1
1999	1523	0.4109	0.1657	0.0290	0.3064	0.3661	0.4612	1
2000	1625	0.3958	0.1710	0.0382	0.2829	0.3458	0.4469	1
2001	1729	0.4162	0.1568	0.0372	0.3140	0.3746	0.4676	1
2002	1860	0.3815	0.1574	0.0426	0.2768	0.3408	0.4386	1
2003	2036	0.3983	0.1549	0.0893	0.2993	0.3592	0.4530	1
2004	2147	0.4072	0.1690	0.0884	0.2913	0.3603	0.4697	1
2005	2158	0.3840	0.1673	0.1104	0.2744	0.3368	0.4385	1
2006	920	0.3766	0.1977	0.1031	0.2413	0.3082	0.4418	1
2007	899	0.4009	0.2008	0.0353	0.2580	0.3402	0.4709	1
2008	901	0.3555	0.2077	0.0278	0.2134	0.2839	0.4220	1

treats deposits as an output. In general, our model produced efficiency scores that differed from the other models.

Figs. 9 and 10 show the time plot of the mean and median efficiency scores obtained from the three alternative models, respectively. Several patterns observed in Fig. 9 are worth mentioning. In general, the model with deposits as an input tends to produce the highest mean and median efficiency scores, while the model with deposits as an output produces the lowest mean and median scores. Our model with deposits as an intermediate product generates efficiency scores that fall between those obtained from the other two alternative models. A potential reason for this is that many bank holding companies may have relatively low deposits. This will tend to lead to higher efficiency scores when treating deposits as an input than when treating deposits as an output.

More importantly, while the efficiency scores seem to be correlated across the models, there are several cases where the model with deposits as an input produces results that are at odds with those produced by the model with deposits as an output. For instance, the model that treats deposits as an input shows a year-to-year decrease in mean efficiency scores in 1987, 1999, 2001, and 2007, while the model that treats deposits as an output shows an increase in mean efficiency scores in those years. Similarly, the mean efficiency scores rise in 1988, 1997, 2000, 2002, 2006, and 2008 according to the model with deposits as an input, while the

Table 4
Descriptive statistics of the efficiency scores obtained from a DEA model that treats deposits as an intermediate product. The sample includes top tier bank holding companies filing consolidated financial statements. Inputs include fixed assets and the number of employees. Outputs include total loans and other earning assets, defined as the sum of securities, federal funds sold, and trading assets. Total deposits are treated as an intermediate product.

Year	*N*	Mean	SD	Minimum	1st quartile	Median	3rd quartile	Maximum
1986	1280	0.5920	0.1524	0.2664	0.4827	0.5692	0.6828	1
1987	1340	0.5562	0.1561	0.1989	0.4460	0.5220	0.6309	1
1988	1366	0.5524	0.1603	0.1381	0.4443	0.5156	0.6186	1
1989	1372	0.6044	0.1543	0.1664	0.4949	0.5715	0.6886	1
1990	1433	0.5358	0.1832	0.1511	0.4050	0.5010	0.6464	1
1991	1447	0.6186	0.1509	0.1860	0.5093	0.5963	0.7057	1
1992	1464	0.5825	0.1712	0.1073	0.4591	0.5627	0.6882	1
1993	1454	0.6432	0.1458	0.1203	0.5468	0.6283	0.7209	1
1994	1190	0.5842	0.1519	0.1654	0.4827	0.5550	0.6495	1
1995	1214	0.5953	0.1502	0.2037	0.4954	0.5682	0.6613	1
1996	1270	0.5539	0.1588	0.2314	0.4485	0.5205	0.6232	1
1997	1373	0.5380	0.1564	0.1434	0.4374	0.5028	0.6063	1
1998	1473	0.4936	0.1608	0.0883	0.3897	0.4573	0.5586	1
1999	1523	0.4835	0.1599	0.0623	0.3771	0.4469	0.5433	1
2000	1625	0.5279	0.1566	0.0821	0.4211	0.4926	0.6057	1
2001	1729	0.5136	0.1536	0.0892	0.4098	0.4801	0.5769	1
2002	1860	0.5023	0.1505	0.0924	0.4003	0.4695	0.5684	1
2003	2036	0.5318	0.1484	0.1203	0.4313	0.5062	0.6059	1
2004	2147	0.5271	0.1526	0.0988	0.4194	0.5030	0.6030	1
2005	2158	0.3984	0.1705	0.1044	0.2844	0.3592	0.4670	1
2006	920	0.4577	0.1822	0.0833	0.3364	0.4173	0.5352	1
2007	899	0.4339	0.2010	0.0897	0.2896	0.3834	0.5222	1
2008	901	0.6135	0.1689	0.1037	0.4957	0.5923	0.7079	1

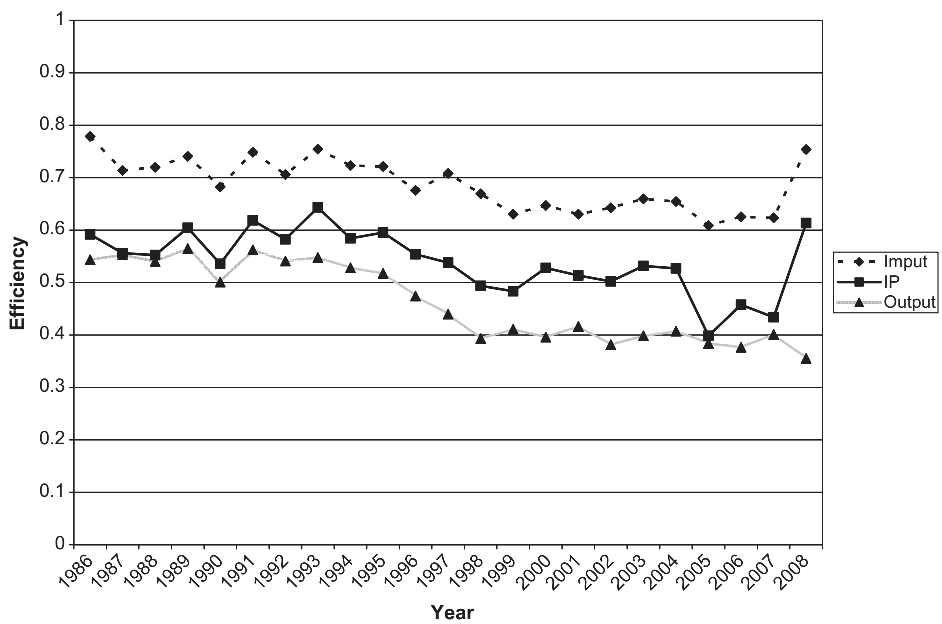

Fig. 9. Mean efficiency scores obtained from three alternative DEA models. The sample includes top tier bank holding companies filing consolidated financial statements. Each model considers fixed assets and the number of employees as inputs, and total loans and other earning assets as outputs. The models differ in their treatment of deposits. The model labeled "Input" treats deposits as an input. The model labeled "Output" treats deposits as an output. The model labeled "IP" treats deposits as an intermediate product in a two-stage production process.

efficiency falls in those years if the model with deposits as an output is utilized. Our model takes the side of the model with deposits as an input in 1987, 1999, 2000, 2001, 2006, 2007, and 2008, while it leans towards the model with deposits as an output in 1988, 1997, and 2002. These findings further demonstrate that the conclusions about the dynamics of bank efficiency may be affected by the researcher's choice of whether to treat deposits as an input or an output. Our model may serve as a unifying framework that captures the bank production process more appropriately, and avoids the "deposit dilemma".

In addition to influencing the estimates of the average efficiency in the entire banking industry, the choice of whether to treat deposits as an input or an output may affect the efficiency-based ranking of the individual banking organization. Table 5 shows the yearly correlations of the efficiency scores for each pair of models. We note that all correlations are positive. However, the correlations between efficiency scores when deposits are treated as an input and those when deposits are treated as an output (Input Out) are below 0.7 in several years indicating that the two models produce divergent ranks. This result is consistent with Hunter and

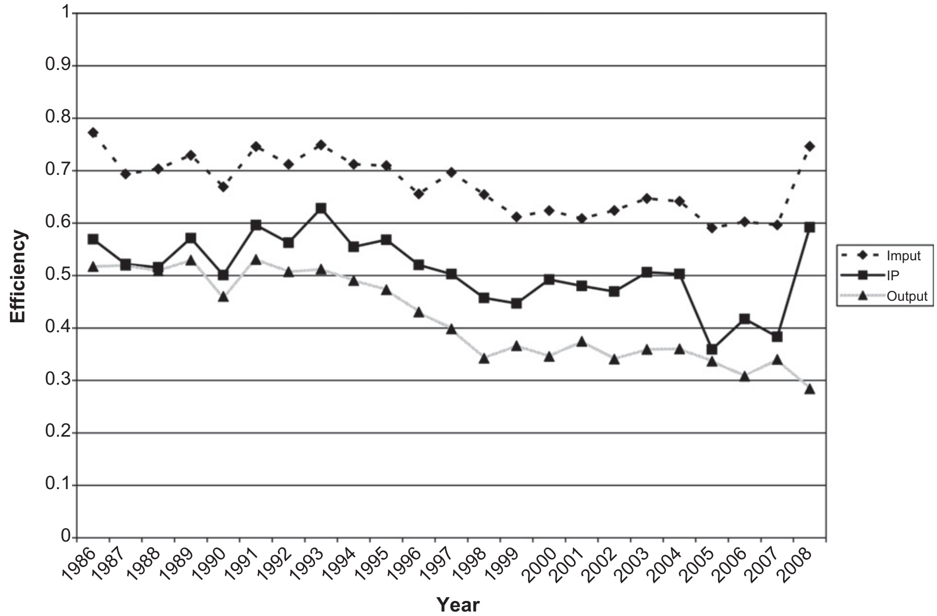

Fig. 10. Median efficiency scores obtained from three alternative DEA models. The sample includes top tier bank holding companies filing consolidated financial statements. Each model considers fixed assets and the number of employees as inputs, and total loans and other earning assets as outputs. The models differ in their treatment of deposits. The model labeled "Input" treats deposits as an input. The model labeled "Output" treats deposits as an output. The model labeled "IP" treats deposits as an intermediate product in a two-stage production process.

Timme (1995), who find the rankings of the individual banks to be weakly correlated across alternative efficiency models.

To summarize, we demonstrate that our model produces reasonable results. Furthermore, the choice of whether to treat deposits as an input or an output matters for the estimated average efficiency in the banking industry as well as for the ranking of individual banking organizations.

Table 5

Correlations of the efficiency scores between the models. The sample includes top tier bank holding companies filing consolidated financial statements. Each model considers fixed assets and the number of employees as inputs, and total loans and other earning assets as outputs. The models differ in their treatment of deposits. The model labeled "Input" treats deposits as an input. The model labeled "Out" treats deposits as an output. The model labeled "IP" treats deposits as an intermediate product in a two-stage production process.

Year	Pearson correlations			Spearman rank correlations		
	Input IP	Out IP	Input Out	Input IP	Out IP	Input Out
1986	0.8573	0.8022	0.5094	0.8134	0.7458	0.3802
1987	0.8975	0.8732	0.7252	0.8281	0.8243	0.6232
1988	0.8748	0.8604	0.6367	0.8067	0.7902	0.4884
1989	0.8699	0.8801	0.6380	0.8188	0.8234	0.5056
1990	0.9292	0.7882	0.6556	0.9394	0.6851	0.5785
1991	0.9184	0.7864	0.5957	0.9153	0.6919	0.4859
1992	0.8906	0.7970	0.5860	0.9216	0.7169	0.5573
1993	0.9630	0.8281	0.7564	0.9686	0.7762	0.7445
1994	0.9439	0.8681	0.7375	0.9214	0.8048	0.6332
1995	0.9602	0.9020	0.8096	0.9503	0.8469	0.7318
1996	0.9500	0.9337	0.8686	0.9238	0.9153	0.8359
1997	0.9455	0.9061	0.8284	0.9181	0.8726	0.7941
1998	0.9723	0.8630	0.8389	0.9661	0.8076	0.8424
1999	0.9751	0.9040	0.8761	0.9761	0.8441	0.8539
2000	0.9841	0.8493	0.8367	0.9812	0.7735	0.7872
2001	0.9829	0.8994	0.8959	0.9791	0.8545	0.8835
2002	0.9816	0.9255	0.9136	0.9787	0.9005	0.9129
2003	0.9877	0.9202	0.9058	0.9884	0.9201	0.9288
2004	0.9800	0.8468	0.7964	0.9783	0.7768	0.7343
2005	0.9311	0.9151	0.8093	0.9039	0.8473	0.7203
2006	0.9591	0.8940	0.8467	0.9572	0.8102	0.8081
2007	0.9460	0.9235	0.8739	0.9304	0.8654	0.8501
2008	0.9603	0.7196	0.6375	0.9734	0.6678	0.6393

6. Conclusion

We develop a DEA model of bank efficiency that treats deposits as an intermediate product in the bank production process. In particular, we recognize that deposits may be considered as either an output or an input, depending on the stage of a bank's production process. As a result, the effect of the amount of deposits on the overall bank efficiency depends on the efficiency at both stages. Since our model does not require a researcher to side with either the production or the intermediation approach, we believe that it has a potential to serve as a unifying framework for bank efficiency estimation.

We apply the modified version of our model to the data, and show that it produces reasonable results. Furthermore, we demonstrate that the choice of whether to treat deposits as an input or an output matters for the estimated average efficiency in the banking industry as well as for the ranking of individual banking organizations. Although the modified version of our model still treats deposits as an intermediate product rather than as an input or an output, it does not allow us to obtain separate efficiency estimates for each stage. Our original model requires disaggregation of inputs in order to get further insights into the bank efficiency at each stage. Such a disaggregation could be an interesting direction for future research.

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