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Decision Support

On the extent analysis method for fuzzy AHP and its applications

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Abstract

In a paper by Chang [D.Y. Chang, Applications of the extent analysis method on fuzzy AHP, European Journal of Operational Research 95 (1996) 649–655], an extent analysis method on fuzzy AHP was proposed to obtain a crisp priority vector from a triangular fuzzy comparison matrix. It is found that the extent analysis method cannot estimate the true weights from a fuzzy comparison matrix and has led to quite a number of misapplications in the literature. In this paper, we show by examples that the priority vectors determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and that the misapplication of the extent analysis method to fuzzy AHP problems may lead to a wrong decision to be made and some useful decision information such as decision criteria and fuzzy comparison matrices not to be considered. We show these problems to avoid any possible misapplications in the future.

Keywords: Analytic hierarchy process; Multiple criteria decision analysis; Fuzzy comparison matrix; Extent analysis

1. Introduction

Analytic hierarchy process (AHP) [20] has been widely used as a useful multiple criteria decision making (MCDM) tool or a weight estimation technique in many areas such as selection, evaluation, planning and development, decision making, forecasting, and so on [24]. The traditional AHP requires crisp judgments. However, due to the complexity and uncertainty involved in real world decision problems, a decision maker (DM) may sometimes feel more confident to provide fuzzy judgments than crisp comparisons.

A number of methods have been developed to handle fuzzy comparison matrices. For example, Van Laarhoven and Pedrycz [25] suggested a fuzzy logarithmic least squares method (LLSM) to obtain triangular

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fuzzy weights from a triangular fuzzy comparison matrix. Wang et al. [29] presented a modified fuzzy LLSM. Buckley [4] utilized the geometric mean method to calculate fuzzy weights. Chang [9] proposed an extent analysis method, which derives crisp weights for fuzzy comparison matrices. Xu [26] brought forward a fuzzy least-squares priority method (LSM). Mikhailov [19] developed a fuzzy preference programming method (PPM), which also derives crisp weights from fuzzy comparison matrices. Csutora and Buckley [10] came up with a Lambda-Max method, which is the direct fuzzification of the well-known $\lambda_{\max}$ method.

Among the above approaches, the extent analysis method has been employed in quite a number of applications [1–3,5–8,11–18,21–23,30] due to its computational simplicity. However, such a method is found unable to derive the true weights from a fuzzy or crisp comparison matrix. The weights determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives at all. Therefore, it should not be used as a method for estimating priorities from a fuzzy pairwise comparison matrix. The purpose of this paper is to show by examples that the priority vectors determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and that the misapplication of the extent analysis method to fuzzy AHP problems may lead to a wrong decision to be made and some useful decision information such as decision criteria and fuzzy comparison matrices not to be considered. We illustrate these problems to avoid any possible misapplications in the future.

The rest of the paper is organized as follows. In Section 2, we briefly review the extent analysis method on fuzzy AHP. In Section 3, three numerical examples are examined using the extent analysis method to show its serious problems and irrationalities. The paper is concluded in Section 4.

2. Review of the extent analysis method on fuzzy AHP

Consider a triangular fuzzy comparison matrix expressed by

$$\widetilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} (1,1,1) & (l_{12},m_{12},u_{12}) & \cdots & (l_{1n},m_{1n},u_{1n}) \\ (l_{21},m_{21},u_{21}) & (1,1,1) & \cdots & (l_{2n},m_{2n},u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1},m_{n1},u_{n1}) & (l_{n2},m_{n2},u_{n2}) & \cdots & (1,1,1) \end{bmatrix}, \tag{1}$$

where $\tilde{a}_{ij} = (l_{ij},m_{ij},u_{ij}) = \tilde{a}_{ji}^{-1} = (1/u_{ji},1/m_{ji},1/l_{ji})$ for $i,j=1,\dots,n$ and $i\neq j$.

To calculate a priority vector of the above triangular fuzzy comparison matrix, Chang [9] suggested an extent analysis method, which is summarized as follows.

Firstly, sum up each row of the fuzzy comparison matrix $\widetilde{A}$ by fuzzy arithmetic operations:

$$\mathrm{RS}_i = \sum_{j=1}^{n} \tilde{a}_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right), \quad i=1,\dots,n. \tag{2}$$

Secondly, normalize the above row sums by

$$\widetilde{S}_i = \frac{\mathrm{RS}_i}{\sum_{j=1}^{n}\mathrm{RS}_j} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} l_{kj}}\right), \quad i=1,\dots,n. \tag{3}$$

Thirdly, compute the degree of possibility of $\widetilde{S}_i \geqslant \widetilde{S}_j$ by the following equation:

$$V(\widetilde{S}_i \geqslant \widetilde{S}_j) = \begin{cases} 1, & \text{if } m_i \geqslant m_j, \\ \frac{u_i-l_j}{(u_i-m_i)+(m_j-l_j)}, & \text{if } l_j \leqslant u_i, \qquad i,j=1,\dots,\ n; j\neq i \\ 0, & \text{others}, \end{cases} \tag{4}$$

where $\widetilde{S}_i = (l_i,m_i,u_i)$ and $\widetilde{S}_j = (l_j,m_j,u_j)$. The definition of possibility degree is shown in Fig. 1.

Fourthly, calculate the degree of possibility of $\widetilde{S}_i$ over all the other $(n-1)$ fuzzy numbers by

$$V\left(\widetilde{S}_i \geqslant \widetilde{S}_j | j=1,\dots,n; j\neq i\right) = \min_{j\in\{1,\dots,n\}, j\neq i} V\left(\widetilde{S}_i \geqslant \widetilde{S}_j\right), \quad i=1,\dots,n. \tag{5}$$

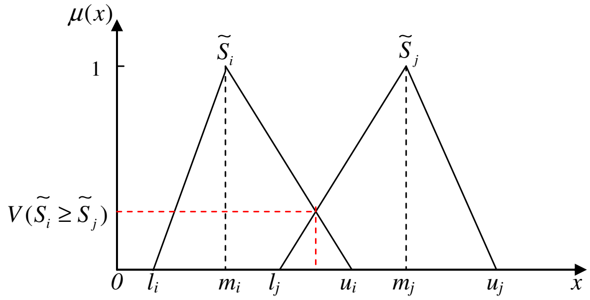

Fig. 1. Definition of the degree of possibility of $V(\widetilde{S}_i \geqslant \widetilde{S}_j)$.

Finally, define the priority vector $W = (w_1, \ldots, w_n)^{\mathrm{T}}$ of the fuzzy comparison matrix $\widetilde{A}$ as

$$w_i = \frac{V\left(\widetilde{S}_i \geqslant \widetilde{S}_j | j = 1, \ldots, n; j \neq i\right)}{\sum_{k=1}^{n} V\left(\widetilde{S}_k \geqslant \widetilde{S}_j | j = 1, \ldots, n; j \neq k\right)}, \quad i = 1, \ldots, n. \tag{6}$$

It must be pointed out that the normalization formula (3) is wrong. The correct normalization formula for a set of triangular fuzzy weights should be as follows:

$$\widetilde{S}_i = \frac{\mathrm{RS}_i}{\sum_{j=1}^{n} \mathrm{RS}_j} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{j=1}^{n} l_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n} u_{ij} + \sum_{k=1,k\neq i}^{n} \sum_{j=1}^{n} l_{kj}} \right), \quad i = 1, \ldots, n. \tag{7}$$

Interested readers are referred to Wang and Elhag [28] for the derivation of this formula and its in-depth discussion.

In the next section, we show by examples that the weights determined by the above extent analysis method do not represent the relative importance of decision criteria or alternatives and cannot be used as their priorities. We also demonstrate that the misapplication of the extent analysis method to fuzzy AHP problems may result in wrong decisions to be made and some fuzzy comparison matrices information to be wasted.

3. Untrue weights and wrong decision making by the extent analysis method

Example 1. Consider two decision criteria with their relative weights being $\tilde{w}_1 = (0.65, 0.7, 0.75)$ and $\tilde{w}_2 = (0.25, 0.3, 0.35)$, based on which the fuzzy comparison matrix on the two criteria can be constructed as

$$\widetilde{A} = \begin{bmatrix} (1,1,1) & (1.8571, 2.333, 3) \\ (0.3333, 0.4286, 0.5385) & (1,1,1) \end{bmatrix},$$

which is a perfectly consistent triangular fuzzy comparison matrix.

By the extent analysis method, we have the following results:

$$\mathrm{RS}_1 = \sum_{j=1}^{n} \tilde{a}_{1j} = (1,1,1) \oplus (1.8571, 2.333, 3) = (2.8571, 3.3333, 4),$$

$$\mathrm{RS}_2 = \sum_{j=1}^{n} \tilde{a}_{2j} = (0.3333, 0.4286, 0.5385) \oplus (1,1,1) = (1.3333, 1.4286, 1.5385),$$

$$\mathrm{RS}_1 \oplus \mathrm{RS}_2 = (2.8571, 3.3333, 4) \oplus (1.3333, 1.4286, 1.5385) = (4.1905, 4.7619, 5.5385),$$

$$\widetilde{S}_1 = \mathrm{RS}_1 \otimes [\mathrm{RS}_1 \oplus \mathrm{RS}_2]^{-1} = (2.8571, 3.3333, 4) \otimes \left(\frac{1}{5.5385}, \frac{1}{4.7619}, \frac{1}{4.1905} \right) = (0.5159, 0.7, 0.9545),$$

$$\begin{aligned}
\widetilde{S}_2 &= \mathrm{RS}_2 \otimes [\mathrm{RS}_1 \oplus \mathrm{RS}_2]^{-1} = (1.3333, 1.4286, 1.5385) \otimes \left(\frac{1}{5.5385}, \frac{1}{4.7619}, \frac{1}{4.1905}\right) \\
&= (0.2407, 0.3, 0.3671), \\
V(\widetilde{S}_1 &\geqslant \widetilde{S}_2) = 1, \\
V(\widetilde{S}_2 &\geqslant \widetilde{S}_1) = 0, \\
w_1 &= \frac{V(\widetilde{S}_1 \geqslant \widetilde{S}_2)}{V(\widetilde{S}_1 \geqslant \widetilde{S}_2) + V(\widetilde{S}_2 \geqslant \widetilde{S}_1)} = 1, \\
w_2 &= \frac{V(\widetilde{S}_2 \geqslant \widetilde{S}_1)}{V(\widetilde{S}_1 \geqslant \widetilde{S}_2) + V(\widetilde{S}_2 \geqslant \widetilde{S}_1)} = 0.
\end{aligned}$$

Accordingly, the priority vector of the two decision criteria are estimated by the extent analysis method as $W = (1, 0)^{\mathrm{T}}$, which means that the second decision criterion is given a zero weight and will not be considered in decision analysis. This is the first problem of the extent analysis method, which is highlighted as follows:

Problem 1. The extent analysis method may assign a zero weight to a decision criterion or alternative, leading to the criterion or alternative not to be considered in decision analysis.

If a decision criterion or alternative may not be considered, then it should be removed from the fuzzy comparison matrix and there is no need to include it in the fuzzy comparison matrix from the very beginning.

The priority vector $W = (1, 0)^{\mathrm{T}}$ derived by the extent analysis method is totally different from the true fuzzy weight vector $\widetilde{W} = ((0.65, 0.7, 0.75), (0.25, 0.3, 0.35))^{\mathrm{T}}$, which clearly shows that w_1 should be within the interval $[0.65, 0.75]$ and w_2 within the interval $[0.25, 0.35]$. However, the weights determined by the extent analysis method both fall outside their intervals and therefore do not represent the relative importance of the two decision criteria. This is the second problem with the extent analysis method, which is highlighted below:

Problem 2. The weights determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and cannot be used as their priorities.

As is known, for a crisp and perfectly consistent pairwise comparison, its priority vector can be generated by summing up its each row and then normalizing its row sums [20]. Suppose this approach could be extended to deal with fuzzy comparison matrices. Then the fuzzy numbers determined by Eq. (3) can be seen as an approximate estimate of the fuzzy weights of the fuzzy comparison matrix defined by Eq. (1). In this sense, $\widetilde{S}_1 = (0.5159, 0.7, 0.9545)$ and $\widetilde{S}_2 = (0.2407, 0.3, 0.3671)$ may approximately represent the relative importance of the two decision criteria, but it is absolutely not the $W = (1, 0)^{\mathrm{T}}$ that stands for the relative importance of the two decision criteria.

Since Eq. (3) is wrong, it should be replaced by Eq. (7). When Eq. (7) is used to normalize RS_1 and RS_2, we have the following results:

$$\widetilde{S}_1 = \mathrm{RS}_1 \otimes [\mathrm{RS}_1 \oplus \mathrm{RS}_2]^{-1} = \left(\frac{2.8571}{2.8571 + 1.5385}, \frac{3.3333}{3.3333 + 1.4286}, \frac{4}{4 + 1.3333}\right) = (0.65, 0.70, 0.75),$$

$$\widetilde{S}_2 = \mathrm{RS}_2 \otimes [\mathrm{RS}_1 \oplus \mathrm{RS}_2]^{-1} = \left(\frac{1.3333}{1.3333 + 4}, \frac{1.4286}{3.3333 + 1.4286}, \frac{1.5385}{1.5385 + 2.8571}\right) = (0.25, 0.30, 0.35),$$

which are exactly the same as the precise fuzzy weights, but this is not always the case, particularly in the situation that fuzzy comparison matrices are not perfectly consistent.

It is easy to see from Eq. (4) that the degree of possibility defined by the extent analysis method is an index for comparing two triangular fuzzy numbers rather than an index for calculating their relative importance. Therefore, normalized degrees of possibility can only show to what degree a triangular fuzzy number is greater than all the others, but cannot be used to represent their relative importance. As far as example 1 is concerned, from the results obtained by the extent analysis method it can only be concluded that the fuzzy weight of the first decision criterion is bigger than that of the second criterion to the degree of 100%, but this absolutely does not mean the weight of the first decision criterion is one while the weight of the second decision criterion is zero.

Example 2. Consider a crisp comparison matrix, as shown below:

$$A = \begin{bmatrix} 1 & 4/3 & 2 & 4 \\ 3/4 & 1 & 3/2 & 3 \\ 1/2 & 2/3 & 1 & 2 \\ 1/4 & 1/3 & 1/2 & 1 \end{bmatrix},$$

which is a perfectly consistent comparison matrix, whose true weight vector can be derived as $W = (0.4, 0.3, 0.2, 0.1)^{\mathrm{T}}$ by any priority method except for the extent analysis method.

Due to the fact that crisp comparison matrices are special cases of triangular fuzzy comparison matrices, the extent analysis method should also be applicable to crisp comparison matrices.

By the extent analysis method, we have the following results for the above crisp comparison matrix:

$$\begin{aligned}
&\mathrm{RS}_1 = 1 + 4/3 + 2 + 4 = 25/3,\\
&\mathrm{RS}_2 = 3/4 + 1 + 3/2 + 3 = 25/4,\\
&\mathrm{RS}_3 = 1/2 + 2/3 + 1 + 2 = 25/6,\\
&\mathrm{RS}_4 = 1/4 + 1/3 + 1/2 + 1 = 25/12,\\
&S_1 = \mathrm{RS}_1 \Big/ \textstyle\sum_{i=1}^{4} \mathrm{RS}_i = 0.4,\\
&S_2 = \mathrm{RS}_2 \Big/ \textstyle\sum_{i=1}^{4} \mathrm{RS}_i = 0.3,\\
&S_3 = \mathrm{RS}_3 \Big/ \textstyle\sum_{i=1}^{4} \mathrm{RS}_i = 0.2,\\
&S_4 = \mathrm{RS}_4 \Big/ \textstyle\sum_{i=1}^{4} \mathrm{RS}_i = 0.1,\\
&V(S_1 \geqslant S_2) = 1, \quad V(S_1 \geqslant S_3) = 1, \quad V(S_1 \geqslant S_4) = 1,\\
&V(S_2 \geqslant S_1) = 0, \quad V(S_2 \geqslant S_3) = 1, \quad V(S_2 \geqslant S_4) = 1,\\
&V(S_3 \geqslant S_1) = 0, \quad V(S_3 \geqslant S_2) = 0, \quad V(S_3 \geqslant S_4) = 1,\\
&V(S_4 \geqslant S_1) = 0, \quad V(S_4 \geqslant S_2) = 0, \quad V(S_4 \geqslant S_3) = 0,\\
&V(S_1 \geqslant S_2, S_1 \geqslant S_3, S_1 \geqslant S_4) = \min(1,1,1) = 1,\\
&V(S_2 \geqslant S_1, S_2 \geqslant S_3, S_1 \geqslant S_4) = \min(0,1,1) = 0,\\
&V(S_3 \geqslant S_1, S_3 \geqslant S_2, S_3 \geqslant S_4) = \min(0,0,1) = 0,\\
&V(S_4 \geqslant S_1, S_4 \geqslant S_2, S_4 \geqslant S_3) = \min(0,0,0) = 0,\\
&W = (w_1, w_2, w_3, w_4)^{\mathrm{T}} = (1,0,0,0)^{\mathrm{T}}.
\end{aligned}$$

It is obvious that such a weight vector also differs from the true weight vector $W = (0.4, 0.3, 0.2, 0.1)^{\mathrm{T}}$ and therefore cannot be used as the relative importance weight vector of the four decision criteria. Here three decision criteria are given an irrational zero weight. If such an untrue weight vector were misused to represent the relative importance of the four decision criteria, then only the first decision criterion would be considered and all the other three decision criteria would be ignored. This is obviously not true. The DM included the four decision criteria in his/her comparison matrix clearly shows that he/she would like to consider all the four decision criteria; otherwise, he/she could construct a reduced comparison matrix or would not construct any comparison matrix at all if only one criterion were to be considered.

Since A is a perfectly consistent comparison matrix, its weight vector can be precisely characterized by $S = (S_1, \ldots, S_4)^{\mathrm{T}} = (0.4, 0.3, 0.2, 0.1)^{\mathrm{T}}$ rather than by $W = (w_1, \ldots w_4)^{\mathrm{T}} = (1, 0, 0, 0)^{\mathrm{T}}$, which only shows the fact that the priority of the first decision criterion is bigger than those of the other three decision criteria to 100% degree, but none of the priorities of the others can be bigger than that of the first decision criterion.

The following example illustrates the key fact that the misapplication of the extent analysis method to fuzzy AHP problems may result in wrong decisions to be made. This is the fundamental problem of the extent analysis method.

Example 3. A big Turkish textile company wishes to make a contract with one catering firm. Alternative Turkish catering firms are *Durusu*, *Mertol* and *Afiyetle*. The goal is to select the best among the three alternatives. The criteria to be considered are hygiene (H), quality of meal (QM), and quality of service (QS), which involve 11 sub-criteria, i.e. hygiene of meal (HM), hygiene of service personnel (HSP), hygiene of service vehicles (HSV), variety of meal (VM), complementary meals in a day (CoM), calorie of meal (CaM), taste of meal (TM), behaviour of service personnel (BSP), service time (ST), communication on phone (CP), and problem solving (PS) ability. Fig. 2 shows the hierarchical structure of the problem. A decision-making group consisting of the customers of the catering firms and five experts is responsible for making comparisons and constructing fuzzy comparison matrices. The results are shown in Tables 1–15. This example and the fuzzy comparison matrices are all taken from [13] with a slight change in Tables 1 and 4.

This problem can be well resolved by using the modified fuzzy LLSM developed in [29], which derives the priorities of the triangular fuzzy comparison matrix in (1) through the solution of the following constrained nonlinear optimization model [29]:

$$\mathrm{Min}\ J = \sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} \left((\ln w_i^L - \ln w_j^U - \ln l_{ij})^2 + (\ln w_i^M - \ln w_j^M - \ln m_{ij})^2 + (\ln w_i^U - \ln w_j^L - \ln u_{ij})^2 \right)$$

$$\text{s.t.} \begin{cases} w_i^L + \sum\limits_{j=1,j\neq i}^{n} w_j^U \geqslant 1, \\ w_i^U + \sum\limits_{j=1,j\neq i}^{n} w_j^L \leqslant 1, \\ \sum\limits_{i=1}^{n} w_i^M = 1, & i = 1, \ldots, n. \\ \sum\limits_{i=1}^{n} (w_i^L + w_i^U) = 2, \\ w_i^U \geqslant w_i^M \geqslant w_i^L > 0, \end{cases} \tag{8}$$

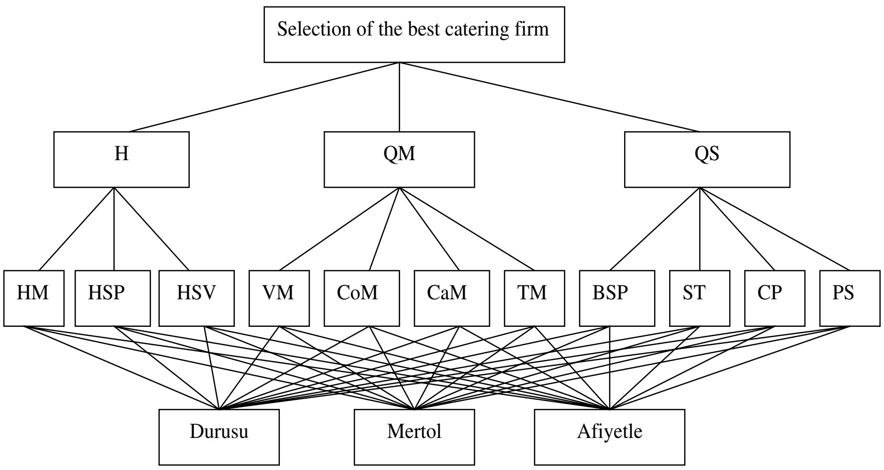

Fig. 2. Hierarchy of catering firm selection problem.

The optimum solution to the above model forms normalized triangular fuzzy weights $\tilde{w}_i = (w_i^L, w_i^M, w_i^U)$, $i = 1, \ldots, n$. Global fuzzy weights can be obtained by solving the following two linear programming models and an equation for each decision alternative A_k $(k = 1, \ldots, K)$ [29]:

$$w_{A_k}^L = \underset{W \in \Omega_W}{\text{Min}} \sum_{j=1}^{m} w_{kj}^L w_j, \quad k = 1, \ldots, K, \tag{9}$$

Table 1
Fuzzy comparison matrix of three decision criteria with respect to the goal and its priority vectors

Criteria	H	QM	QS	Priority vector	
				Extent analysis	Modified fuzzy LLSM
H	(1, 1, 1)	(2/3, 1, 3/2)	(2/7, 1/3, 2/5)	0	(0.1781, 0.2098, 0.2477)
QM	(2/3, 1, 3/2)	(1, 1, 1)	(2/5, 1/2, 2/3)	0	(0.1933, 0.2402, 0.3015)
QS	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(1, 1, 1)	1	(0.5203, 0.5499, 0.5590)

Table 2
Fuzzy comparison matrix of three sub-criteria with respect to hygiene and its priority vectors

Sub-criteria	HM	HSP	HSV	Priority vector	
				Extent analysis	Modified fuzzy LLSM
HM	(1, 1, 1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)	0.70	(0.4615, 0.5, 0.5128)
HSP	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/3, 1, 3/2)	0.15	(0.2051, 0.25, 0.3077)
HSV	(2/5, 1/2, 2/3)	(2/3, 1, 3/2)	(1, 1, 1)	0.15	(0.2051, 0.25, 0.3077)

Table 3
Fuzzy comparison matrix of four sub-criteria with respect to quality of meal and its priority vectors

Sub-criteria	VM	CoM	CaM	TM	Priority vector	
					Extent analysis	Modified fuzzy LLSM
VM	(1, 1, 1)	(3/2, 2, 5/2)	(2/7, 1/3, 2/5)	(5/2, 3, 7/2)	0.19	(0.2188, 0.2488, 0.2783)
CoM	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/7, 1/3, 2/5)	(7/2, 4, 9/2)	0.05	(0.1729, 0.1890, 0.2107)
CaM	(5/2, 3, 7/2)	(5/2, 3, 7/2)	(1, 1, 1)	(5/2, 3, 7/2)	0.76	(0.4371, 0.4768, 0.5095)
TM	(2/7, 1/3, 2/5)	(2/9, 1/4, 2/7)	(2/7, 1/3, 2/5)	(1, 1, 1)	0	(0.0815, 0.0854, 0.0911)

Table 4
Fuzzy comparison matrix of four sub-criteria with respect to quality of service and its priority vectors

Sub-criteria	BSP	ST	CP	PS	Priority vector	
					Extent analysis	Modified fuzzy LLSM
BSP	(1, 1, 1)	(2/9, 1/4, 2/7)	(7/2, 4, 9/2)	(2/7, 1/3, 2/5)	0	(0.1408, 0.1493, 0.1597)
ST	(7/2, 4, 9/2)	(1, 1, 1)	(7/2, 4, 9/2)	(5/2, 3, 7/2)	0.99	(0.4826, 0.5173, 0.5472)
CP	(2/9, 1/4, 2/7)	(2/9, 1/4, 2/7)	(1, 1, 1)	(2/7, 1/3, 2/5)	0	(0.0707, 0.0747, 0.0802)
PS	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)	(5/2, 3, 7/2)	(1, 1, 1)	0.01	(0.2321, 0.2587, 0.2866)

Table 5
Fuzzy comparison matrix of three catering firms with respect to hygiene of meal and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(5/2, 3, 7/2)	(3/2, 2, 5/2)	0.66	(0.4668, 0.5278, 0.5760)
Mertol	(2/7, 1/3, 2/5)	(1, 1, 1)	(2/7, 1/3, 2/5)	0	(0.1343, 0.1396, 0.1476)
Afiyetle	(2/5, 1/2, 2/3)	(5/2, 3, 7/2)	(1, 1, 1)	0.34	(0.2897, 0.3325, 0.3857)

Table 6
Fuzzy comparison matrix of three catering firms with respect to hygiene of service personnel and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(2/3, 1, 3/2)	(2/9, 1/4, 2/7)	0	(0.1592, 0.1840, 0.2127)
Mertol	(2/3, 1, 3/2)	(1, 1, 1)	(2/5, 1/2, 2/3)	0	(0.1867, 0.2318, 0.2906)
Afiyetle	(7/2, 4, 9/2)	(3/2, 2, 5/2)	(1, 1, 1)	1	(0.5502, 0.5842, 0.6006)

Table 7
Fuzzy comparison matrix of three catering firms with respect to hygiene of service vehicles and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(2/3, 1, 3/2)	(2/7, 1/3, 2/5)	0	(0.1781, 0.2098, 0.2477)
Mertol	(2/3, 1, 3/2)	(1, 1, 1)	(2/5, 1/2, 2/3)	0	(0.1933, 0.2402, 0.3015)
Afiyetle	(5/2, 3, 7/2)	(3/2, 2, 5/2)	(1, 1, 1)	1	(0.5203, 0.5499, 0.5590)

Table 8
Fuzzy comparison matrix of three catering firms with respect to variety of meal and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(2/7, 1/3, 2/5)	(2/3, 1, 3/2)	0	(0.1805, 0.2223, 0.2670)
Mertol	(5/2, 3, 7/2)	(1, 1, 1)	(1, 1, 1)	0.97	(0.4566, 0.4566, 0.4566)
Afiyetle	(2/3, 1, 3/2)	(1, 1, 1)	(1, 1, 1)	0.03	(0.2764, 0.3211, 0.3629)

Table 9
Fuzzy comparison matrix of three catering firms with respect to complementary meals in a day and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(5/2, 3, 7/2)	(2/3, 1, 3/2)	0.87	(0.3967, 0.4602, 0.5144)
Mertol	(2/7, 1/3, 2/5)	(1, 1, 1)	(1, 1, 1)	0	(0.2207, 0.2207, 0.2207)
Afiyetle	(2/3, 1, 3/2)	(1, 1, 1)	(1, 1, 1)	0.13	(0.2648, 0.3190, 0.3825)

Table 10
Fuzzy comparison matrix of three catering firms with respect to calorie of meal and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(2/9, 1/42/7)	(2/7, 1/3, 2/5)	0	(0.1130, 0.1207, 0.1313)
Mertol	(7/2, 4, 9/2)	(1, 1, 1)	(2/7, 1/3, 2/5)	0.31	(0.2794, 0.3043, 0.3341)
Afiyetle	(5/2, 3, 7/2)	(5/2, 3, 7/2)	(1, 1, 1)	0.69	(0.5346, 0.5750, 0.6075)

Table 11
Fuzzy comparison matrix of three catering firms with respect to taste of meal and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1, 1, 1)	(2/3, 1, 3/2)	(1, 1, 1)	0	(0.2731, 0.3126, 0.3489)
Mertol	(2/3, 1, 3/2)	(1, 1, 1)	(2/9, 1/4, 2/7)	0	(0.1604, 0.1966, 0.2361)
Afiyetle	(1, 1, 1)	(7/2, 4, 9/2)	(1, 1, 1)	1	(0.4908, 0.4908, 0.4908)

Table 12
Fuzzy comparison matrix of three catering firms with respect to behavior of service personnel and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1,1,1)	(7/2,4,9/2)	(5/2,3,7/2)	1	(0.6110,0.6337,0.6489)
Mertol	(2/9,1/4,2/7)	(1,1,1)	(1,1,1)	0	(0.1705,0.1744,0.1804)
Afiyetle	(2/7,1/3,2/5)	(1,1,1)	(1,1,1)	0	(0.1797,0.1919,0.2086)

Table 13
Fuzzy comparison matrix of three catering firms with respect to service time and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1,1,1)	(3/2,2,5/2)	(2/7,1/3,2/5)	0.05	(0.2443,0.2827,0.3291)
Mertol	(2/5,1/2,2/3)	(1,1,1)	(7/2,4,9/2)	0.64	(0.3729,0.4142,0.4614)
Afiyetle	(5/2,3,7/2)	(2/9,1/4,2/7)	(1,1,1)	0.31	(0.2904,0.2987,0.3018)

Table 14
Fuzzy comparison matrix of three catering firms with respect to communication on phone and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1,1,1)	(7/2,4,9/2)	(2/3,1,3/2)	0.86	(0.4242,0.4934,0.5573)
Mertol	(2/9,1/4,2/7)	(1,1,1)	(2/3,1,3/2)	0	(0.1782,0.1958,0.2098)
Afiyetle	(2/3,1,3/2)	(2/3,1,3/2)	(1,1,1)	0.14	(0.2329,0.3108,0.3976)

Table 15
Fuzzy comparison matrix of three catering firms with respect to problem solving ability and its priority vectors

Catering firms	Durusu	Mertol	Afiyetle	Priority vector	
				Extent analysis	Modified fuzzy LLSM
Durusu	(1,1,1)	(1,1,1)	(2/9,1/4,2/7)	0	(0.1714,0.1769,0.1769)
Mertol	(1,1,1)	(1,1,1)	(2/7,1/3,2/5)	0	(0.1782,0.1922,0.2101)
Afiyetle	(7/2,4,9/2)	(5/2,3,7/2)	(1,1,1)	1	(0.6130,0.6309,0.6505)

$$w_{A_k}^U = \underset{W\in\Omega_W}{\mathrm{Max}} \sum_{j=1}^{m} w_{kj}^U w_j, \quad k = 1,\dots,K, \tag{10}$$

$$w_{A_k}^M = \sum_{j=1}^{m} w_{kj}^M w_j^M, \quad k = 1,\dots,K, \tag{11}$$

where $\Omega_W = \left\{W = (w_1,\dots,w_m)^{\mathrm{T}} | w_j^L \leqslant w_j \leqslant w_j^U, \sum_{j=1}^m w_j = 1, j = 1,\dots,m\right\}$ is the space of weights, (w_j^L, w_j^M, w_j^U) is the normalized triangular fuzzy weight of criterion j ($j = 1,\dots,m$) and $(w_{kj}^L, w_{kj}^M, w_{kj}^U)$ is the normalized triangular fuzzy weight of alternative A_k with respect to the criterion j ($k = 1,\dots,K$; $j = 1,\dots,m$).

The modified fuzzy LLSM produces more precise fuzzy weights with narrower support intervals than the fuzzy LLSM in [27]. Take examples 1 and 2 for instance, the modified fuzzy LLSM derives the precise weight vectors $\widetilde{W} = ((0.65, 0.7, 0.75), (0.25, 0.3, 0.35))^{\mathrm{T}}$ and $W = (0.4, 0.3, 0.2, 0.1)^{\mathrm{T}}$ for them. The local and global weights for example 3 obtained by the modified fuzzy LLSM and the extent analysis method are presented in Tables 1–17 and Fig. 3. It is easily observed from Table 17 and Fig. 3 that the modified fuzzy LLSM evaluates *Afiyetle* as the best catering firm. However, the extent analysis method selects *Mertol* which is evaluated as the worst by the modified fuzzy LLSM as the best catering firm, as shown in Table 16. This is a fundamental problem of the extent analysis method, which is highlighted as follows:

Table 16
Synthesis of local priority vectors by the extent analysis method

Local weights of three catering firms with respect to hygiene					
	HM	HSP	HSV		Local weights
Weight	0.70	0.15	0.15		
Durusu	0.66	0	0		0.462
Mertol	0	0	0		0
Afiyetle	0.34	1	1		0.538
Local weights of three catering firms with respect to quality of meal					
	VM	CoM	CaM	TM	Local weights
Weight	0.19	0.05	0.76	0	
Durusu	0	0.87	0	0	0.044
Mertol	0.97	0	0.31	0	0.420
Afiyetle	0.03	0.13	0.69	1	0.537
Local weights of three catering firms with respect to quality of service					
	BSP	ST	CP	PS	Local weights
Weight	0	0.99	0	0.01	
Durusu	1	0.05	0.86	0	0.050
Mertol	0	0.64	0	0	0.634
Afiyetle	0	0.31	0.14	1	0.317
Global weights of three catering firms with respect to the goal					
	H	QM	QS		Global weights
Weight	0	0	1		
Durusu	0.462	0.044	0.050		0.050
Mertol	0	0.420	0.634		0.634
Afiyetle	0.538	0.537	0.317		0.317

Problem 3. The extent analysis method may make a wrong decision and select the worst decision alternative as the best one when it is misused for solving a fuzzy AHP problem.

The reason for the extent analysis method to make a wrong decision here is because it assigns all weight of unity to the QS (quality of service) criterion and ignores the other two decision criteria H (hygiene) and QM (quality of meal). In other words, the hierarchical structure in Fig. 2 is oversimplified by the extent analysis method and the two decision criteria H and QM including eight sub-criteria in total are both removed from decision analysis. In this situation, it is no wonder that the extent analysis method makes a wrong decision and selects the worst alternative as the best.

The removal of the two decision criteria and eight sub-criteria from decision analysis also makes the construction of the fuzzy comparison matrices relating to theses criteria and sub-criteria become redundant. More specifically, the fuzzy comparison matrices in Tables 3, 4 and Tables 8–15 all become redundant and there is no need to construct them at all when the extent analysis method is utilized to solve the problem. All the efforts made by the decision-making group in constructing these fuzzy comparison matrices have gone to waste. This waste of information is usually not allowed and also unacceptable in decision analysis. This is the fourth problem caused by the extent analysis method, which is highlighted as follows.

Problem 4. The extent analysis method cannot make full use of all the fuzzy comparison matrices information and may cause some useful fuzzy comparison matrices information to be wasted when it assigns an irrational zero weight to some useful decision criteria or sub-criteria.

The impact of zero weights on decision analysis and the final decision result not only makes the fuzzy comparison matrices in Tables 3, 4 and Tables 8–15 redundant, but also makes the computation of these fuzzy comparison matrices become unnecessary. That is to say, the calculations performed by the extent analysis

Table 17
Synthesis of local priority vectors by the modified fuzzy LLSM

Local weights of three catering firms with respect to hygiene

	HM	HSP	HSV	Local weights
Weight	(0.4615, 0.5, 0.5128)	(0.2051, 0.25, 0.3077)	(0.2051, 0.25, 0.3077)	
Durusu	(0.4668, 0.5278, 0.5760)	(0.1592, 0.1840, 0.2127)	(0.1781, 0.2098, 0.2477)	(0.3055, 0.3624, 0.4089)
Mertol	(0.1343, 0.1396, 0.1476)	(0.1867, 0.2318, 0.2906)	(0.1933, 0.2402, 0.3015)	(0.1612, 0.1878, 0.2280)
Afiyetle	(0.2897, 0.3325, 0.3857)	(0.5502, 0.5842, 0.6006)	(0.5203, 0.5499, 0.5590)	(0.4082, 0.4498, 0.4919)

Local weights of three catering firms with respect to quality of meal

	VM	CoM	CaM	TM	Local weights
Weight	(0.2188, 0.2488, 0.2783)	(0.1729, 0.1890, 0.2107)	(0.4371, 0.4768, 0.5095)	(0.0815, 0.0854, 0.0911)	
Durusu	(0.1805, 0.2223, 0.2670)	(0.3967, 0.4602, 0.5144)	(0.1130, 0.1207, 0.1313)	(0.2731, 0.3126, 0.3489)	(0.1910, 0.2265, 0.2673)
Mertol	(0.4566, 0.4566, 0.4566)	(0.2207, 0.2207, 0.2207)	(0.2794, 0.3043, 0.3341)	(0.1604, 0.1966, 0.2361)	(0.2950, 0.3172, 0.3406)
Afiyetle	(0.2764, 0.3211, 0.3629)	(0.2648, 0.3190, 0.3825)	(0.5346, 0.5750, 0.6075)	(0.4908, 0.4908, 0.4908)	(0.4043, 0.4563, 0.5027)

Local weights of three catering firms with respect to quality of service

	BSP	ST	CP	PS	Local weights
Weight	(0.1408, 0.1493, 0.1597)	(0.4826, 0.5173, 0.5472)	(0.0707, 0.0747, 0.0802)0	(0.2321, 0.2587, 0.2866)	
Durusu	(0.6110, 0.6337, 0.6489)	(0.2443, 0.2872, 0.3291)	(0.4242, 0.4934, 0.5573)	(0.1714, 0.1769, 0.1769)	(0.2878, 0.3258, 0.3631)
Mertol	(0.1705, 0.1744, 0.1804)	(0.3729, 0.4142, 0.4614)	(0.1782, 0.1958, 0.2098)	(0.1782, 0.1922, 0.2101)	(0.2709, 0.3047, 0.3434)
Afiyetle	(0.1719, 0.1919, 0.2086)	(0.2904, 0.2987, 0.3018)	(0.2329, 0.3108, 0.3976)	(0.6130, 0.6309, 0.6505)	(0.3430, 0.3696, 0.3963)

Global weights of three catering firms with respect to the goal

	H	QM	QS	Global weights
Weight	(0.1781, 0.2098, 0.2477)	(0.1933, 0.2402, 0.3015)	(0.5203, 0.5499, 0.5590)	
Durusu	(0.3055, 0.3624, 0.4089)	(0.1910, 0.2265, 0.2673)	(0.1910, 0.2265, 0.2673)	(0.2618, 0.3096, 0.3559)
Mertol	(0.1612, 0.1878, 0.2280)	(0.2950, 0.3172, 0.3406)	(0.2950, 0.3172, 0.3406)	(0.2484, 0.2831, 0.3221)
Afiyetle	(0.4082, 0.4498, 0.4919)	(0.4043, 0.4563, 0.5027)	(0.4043, 0.4563, 0.5027)	(0.3707, 0.4072, 0.5554)

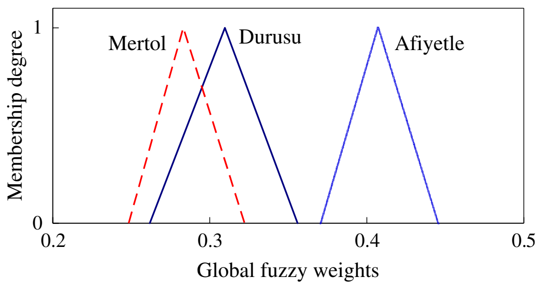

Fig. 3. Global fuzzy weights of three catering firms obtained by the modified fuzzy LLSM.

method in Tables 3, 4 and Tables 8–15 are in fact redundant. No matter what values these calculations get, the final decision conclusion will not be affected. Therefore, there is no need to perform an extent analysis for these fuzzy comparison matrices at all.

From the examination of the three numerical examples, we now come to the conclusion that the extent analysis method is not a method for deriving priorities from a fuzzy comparison matrix and may assign an irrational zero weight to some useful decision criteria and sub-criteria, leading to them not to be considered, the fuzzy comparison matrices information related to these criteria and sub-criteria to be wasted, and a wrong decision to be made. The weights determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and cannot be used as their priorities.

4. Conclusions

In this paper the extent analysis method on fuzzy AHP was re-examined with three numerical examples. It was shown that

- The extent analysis method might assign an irrational zero weight to some useful decision criteria and sub-criteria, leading to them not to be considered in decision analysis.
- The extent analysis method could not make full use of all the fuzzy comparison matrices information and might cause some useful fuzzy comparison matrices information to be wasted when it assigns an irrational zero weight to some useful decision criteria or sub-criteria.
- The weights determined by the extent analysis method do not represent the relative importance of decision criteria or alternatives and could not be used as their priorities.
- The extent analysis method might make a wrong decision and select the worst decision alternative as the best one when it was misused for solving a fuzzy AHP problem.

Based on these pieces of evidence, we came to the conclusion that the extent analysis method is not a method for deriving priorities from a fuzzy comparison matrix. It is a method for showing to what degree the priority of one decision criterion or alternative is bigger than those of all the others in a fuzzy comparison matrix. Since the use of the extent analysis method for solving fuzzy AHP problems may result in a wrong decision to be made, misapplications should be avoided.

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References

[1] F.T. Bozbura, A. Beskese, Prioritization of organizational capital measurement indicators using fuzzy AHP, International Journal of Approximate Reasoning 44 (2007) 124–147.
[2] F.T. Bozbura, A. Beskese, C. Kahraman, Prioritization of human capital measurement indicators using fuzzy AHP, Expert Systems with Applications 32 (2007) 1100–1112.
[3] C.E. Bozdağ, C. Kahraman, D. Ruan, Fuzzy group decision making for selection among computer integrated manufacturing systems, Computers in Industry 51 (2003) 13–29.
[4] J.J. Buckley, Fuzzy hierarchical analysis, Fuzzy Sets and Systems 17 (1985) 233–247.
[5] G. Büyüközkan, Multi-criteria decision making for e-marketplace selection, Internet Research 14 (2) (2004) 139–154.
[6] G. Büyüközkan, T. Ertay, C. Kahraman, D. Ruan, Determining the importance weights for the design requirements in the house of quality using the fuzzy analytic network approach, International Journal of Intelligent Systems 19 (2004) 443–461.
[7] G. Büyüközkan, C. Kahraman, D. Ruan, A fuzzy multi-criteria decision approach for software development strategy selection, International Journal of General Systems 33 (2004) 259–280.
[8] F.T.S. Chan, N. Kumar, Global supplier development considering risk factors using fuzzy extended AHP-based approach, Omega 35 (2007) 417–431.
[9] D.Y. Chang, Applications of the extent analysis method on fuzzy AHP, European Journal of Operational Research 95 (1996) 649–655.
[10] R. Csutora, J.J. Buckley, Fuzzy hierarchical analysis: The Lamda-Max method, Fuzzy Sets and Systems 120 (2001) 181–195.
[11] T. Ertay, G. Büyüközkan, C. Kahraman, D. Ruan, Quality function deployment implementation based on analytic network process with linguistic data: An application in automotive industry, Journal of Intelligent and Fuzzy Systems 16 (2005) 221–232.
[12] Y.C. Erensal, T. Öncan, M.L. Demircan, Determining key capabilities in technology management using fuzzy analytic hierarchy process: A case study of Turkey, Information Sciences 176 (2006) 2755–2770.
[13] C. Kahraman, U. Cebeci, D. Ruan, Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey, International Journal of Production Economics 87 (2004) 171–184.
[14] C. Kahraman, U. Cebeci, Z. Ulukan, Multi-criteria supplier selection using fuzzy AHP, Logistics Information Management 16 (6) (2003) 382–394.
[15] C. Kahraman, T. Ertay, G. Büyüközkan, A fuzzy optimization model for QFD planning process using analytic network approach, European Journal of Operational Research 171 (2006) 390–411.

[16] C. Kahraman, D. Ruan, I. Doğan, Fuzzy group decision-making for facility location selection, Information Sciences 157 (2003) 135–153.

[17] O. Kulak, C. Kahraman, Fuzzy multi-attribute selection among transportation companies using axiomatic design and analytic hierarchy process, Information Sciences 170 (2005) 191–210.

[18] C.K. Kwong, H. Bai, Determining the importance weights for the customer requirements in QFD using a fuzzy AHP with an extent analysis approach, IIE Transactions 35 (2003) 619–626.

[19] L. Mikhailov, Deriving priorities from fuzzy pairwise comparison judgments, Fuzzy Sets and Systems 134 (2003) 365–385.

[20] T.L. Saaty, Multicriteria Decision Making: The Analytic Hierarchy Process, RWS Publications, Pittsburgh, PA, 1988.

[21] Y.C. Tang, M. Beynon, Application and development of a fuzzy analytic hierarchy process within a capital investment study, Journal of Economics and Management 1 (2005) 207–230.

[22] E. Tolga, M.L. Demircan, C. Kahraman, Operating system selection using fuzzy replacement analysis and analytic hierarchy process, International Journal of Production Economics 97 (2005) 89–117.

[23] F. Tüysüz, C. Kahraman, Project risk evaluation using a fuzzy analytic hierarchy process: An application to information technology projects, International Journal of Intelligent Systems 21 (6) (2006) 559–584.

[24] O.S. Vaidya, S. Kumar, Analytic hierarchy process: An overview of applications, European Journal of Operational Research 169 (2006) 1–29.

[25] P.J.M. Van Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets and Systems 11 (1983) 229–241.

[26] R. Xu, Fuzzy least-squares priority method in the analytic hierarchy process, Fuzzy Sets and Systems 112 (2000) 359–404.

[27] P.J.M. van Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets and Systems 11 (1983) 229–241.

[28] Y.M. Wang, T.M.S. Elhag, On the normalization of interval and fuzzy weights, Fuzzy Sets and Systems 157 (2006) 2456–2471.

[29] Y.M. Wang, T.M.S. Elhag, Z.S. Hua, A modified fuzzy logarithmic least squares method for fuzzy analytic hierarchy process, Fuzzy Sets and Systems 157 (2006) 3055–3071.

[30] K.J. Zhu, Y. Jing, D.Y. Chang, A discussion on Extent Analysis Method and application of fuzzy AHP, European Journal of Operational Research 116 (1999) 450–456.