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Multi-Scale Approximate Entropy Analysis of Foreign Exchange Markets Efficiency

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Abstract

Market efficiency analysis is an important aspect in financial engineering.Based on weak-form efficient markets hypothesis (EMH), we characterize the market efficiency in foreign exchange (FX) markets by using the multi-scale approximate entropy (MApEn) to assess the randomness in FX markets. We split 17 daily FX rates from 1984 to 2011 into there periods by two global events, Southeast Asia currency crisis and American sub-prime crisis. The empirical results indicate that the developed FX markets is more efficient than emerging FX markets, and that the financial crisis promotes the market efficiency in FX markets significantly, especially in emerging markets, like China, Hong Kong, Korea and African market.

Keywords: Financial engineering; Entropy; Approximate entropy (ApEn); Multi-scale ApEn; Market efficiency;

1. Introduction

Suffering from the financial crisis started in 2008, the global monetary system has made a significant change, which spurs more attentions on the efficiency and complexity of foreign exchange markets [1-4]. Although the efficient-market hypothesis (EMH) has become controversial because of substantial and lasting inefficiencies are approved in empirical literatures, it is still an important starting point to study market efficiency. Following the innovative work of Fama [5], the financial market efficiency can be properly described by dynamic complexity or randomness, which is widely measured by entropy developed in econophysics (such as approximate entropy [3, 4, 6], cross-sample entropy [7, 8], information entropy [9-11] and Lempel-Ziv complexity [12]).

Entropy, a concept from statistical physics, is broadly used to quantify the disorder and uncertainty of complex dynamic systems. Considering the inherent non-linearity and noise of time series in physiology and medicine financial, Pincus [13] proposed the approximate entropy (ApEn) method based on information theory, and then applied to describe financial time series as a useful measurement of system stability, with rapid increases possibility on predicting remarkable changes in a financial variable [6]. In 2007, the ApEn method has been used to quantify the FX rate time series as a reliable estimator of the market efficiency in FX markets, and shows that the ApEn values for European and North American FX markets are generally larger than those for African and Asian ones except

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Japan [3]. Eom et al [4] employed a symbolic transfer entropy and ApEn method to study information flows in FX rates and found a nearly positive relationship between the difference in the degree of asymmetry of information flows and the difference in the degree of market efficiency. For further characterizing the complexity, a multi-scale entropy (MSE) was introduced for complex time series and widely used in physiologic and biological time series [14, 15]. Moreover, Martina, et al [16] applied the MSE method to analyze the dynamical properties of crude oil price, the results of which shown the MSE approach can shed light on the structure of crude oil markets as well as on its link to macroeconomic conditions and socio-political extreme events.

Based on the empirical studies of ApEn and MSE methods, a multi-scale entropy (named as multi-scale ApEn or MApEn) approach is introduced in this paper. And our motivation is to characterize the market efficiency in FX markets and analyze the market reactions to global financial crisis, such as the Southeast Asia currency crisis and American sub-prime crisis, by the proposed MApEn measurement. The empirical result shows that the MApEn approach outperforms the ApEn method and overcomes the shortcoming of that the ApEn portrays the market efficiency merely on single scale. And we also find solid empirical evidences of that the financial crisis increases the market efficiency of FX markets considerably in general, especially of emerging markets.

The remaining is organized as follows. The next section presents the MApEn method which used to analysis the FX rate time series. Section 3 presents the EMH based on the MApEn method. Then, Section 4 gives the data statistics and the main empirical results. Finally, Section 5 concludes the paper.

2. Methodology

2.1. Approximate entropy (ApEn)

To analysis the short and noisy time series, Pincus [13] proposed the approximate entropy (ApEn) family of parameters, which have been wildly used to quantify the randomness inherent in physiology, medicine and finance time series. The algorithm for ApEn computation can be defined as follows:

Let x={x(1),x(2),...,x(N)} be the length-N time series. Fix input parameters m and r, where m is the embedding dimension, and r is the tolerance for accepting matches.

- Step 1 Create embedding vectors X(i), each is made up of m consecutive values of x,

$$\boldsymbol{X}(i)=\left[x(i),x(i+1),\cdots,x(i+m-1)\right],\qquad 1\le i\le N-m+1\,. \tag{1}$$

- Step 2 The distance d[X(i), X(j)] between two vectors X(i) and X(j) is defined by

$$d\left[\boldsymbol{X}(i),\boldsymbol{X}(j)\right]=\max_{k=0,\cdots,m-1}\left\{\left|x(i+k)-x(j+k)\right|\right\}. \tag{2}$$

- Step 3 For each i, $1\le i\le N-m+1$, the probability that any X(j) is within r of X(i)

$$B_i^m(r)=\frac{\text{mumber of } 1\le j\le N-m \text{ such that } d\left[\boldsymbol{X}(i),\boldsymbol{X}(j)\right]\le r}{N-m+1}\,, \tag{3}$$

where, the vector X(i) is so-called template, which is matched if a X(j) is within r of X(i). And then the average of $B_i^m(r)$ is defined by

$$\boldsymbol{B}^m(r)=\frac{1}{N-m+1}\sum_{i=1}^{N-m}\ln B_i^m(r)\,. \tag{4}$$

- Step 4 Increase the embedding dimension to m+1, and repeat the Step 1 to Step 3. We have

$$\boldsymbol{B}^{m+1}(r)=\frac{1}{N-m}\sum_{i=1}^{N-m-1}\ln B_i^{m+1}(r). \tag{5}$$

- Step 5 $\boldsymbol{B}^{m}(r)$ and $\boldsymbol{B}^{m+1}(r)$ denote the probabilities of template matching for m and m+1 points respectively. Therefore, the ApEn is defined by

$$ApEn(m,r)=\lim_{N\to\infty}\left\{\boldsymbol{B}^{m}(r)-\boldsymbol{B}^{m+1}(r)\right\}. \tag{6}$$

In practice, N is a finite scalar, so the ApEn is estimated by the following statistic,

$$ApEn(m,r,N)=\boldsymbol{B}^{m}(r)-\boldsymbol{B}^{m+1}(r). \tag{7}$$

It can be seen that the ApEn value varies depending on the different embedding dimension m and similar tolerance r. As argued by Pincus [13], m=1, 2 and r=0.1SD~0.25SD are widely used and validated, where SD is the standard deviation of time series. Hence, we preset the embedding dimension m=2 and similar tolerance r=0.25SD.

2.2. Multi-scale approximate entropy (MApEn)

The ApEn value, first order difference of the probability of template matching depends on single scale analysis but not account for characteristics related to system structure more than the shortest scale. To overcome these limitations, the MApEn method is proposed to reflect the financial system complexity under different scales. The algorithm for MApEn computation is defined as follows [14, 15]:

- Step 1 Given a one-dimensional length-N discrete time series x={x(1),x(2),...,x(N)}.
- Step 2 Construct consecutive coarse-grained series determined by the scale factor τ. When the scale factor τ=1, the coarse-grained series is as the same as the original series; When τ>1, the coarse-grained series yτ={yτ(1),yτ(2),..., yτ(N/τ)} is defined by

$$y^{\tau}(j)=\frac{1}{\tau}\sum_{i=(j-1)\tau}^{j\tau}x(i),\quad 1\le j\le N/\tau. \tag{8}$$

- Step 3 Calculate the ApEn value for each coarse-grained series plotted as a function of the scale factor τ, which can get the value of MApEn(τ).

In the MApEn method, the similar tolerance r is set at a certain percentage, 25%, of the standard deviation of original series, and firm for all scales in order to control the effects from the SDs of coarse-grained series under different scales. And we do not infer to this complex relationship between ApEn and SD in this paper as the limited context.

3. Efficiency market hypothesis based on MApEn

3.1. Efficient markets hypothesis (EMH)

The efficiency we assessed in this work refers to the weak-form EMH, which assumes that all the prices already reflect all information contained in the history of past trading [21]. In other words, the actual price will be a good estimate of its intrinsic value at any time, consequently all investors cannot use technical analysis on the historical trading data to forecast the future price and get excess profit. Therefore, the prices in efficient FX markets are should be following a random walk process. Let Pt and Pt-1 denote the daily exchange rate at time t and t-1 respectively, and their logarithmic are defined by

$$p_t=\ln P_t,\ \ p_{t-1}=\ln P_{t-1}. \tag{9}$$

Then, both price pt and pt-1 should satisfy the random walk model as follows:

$$p_t = p_{t-1} + \varepsilon_t \ , \tag{10}$$

where $\{\varepsilon_t\}$ is the independent and identically distributed white noise sequence, i.e., $E(\varepsilon_t)=0$, $\mathrm{Var}(\varepsilon_t)= E(\varepsilon_t^2) =\sigma^2<\infty$.

Similar to Oh, et al [4], the logarithmic return of exchange rate r_t and the normalized logarithmic return R_t are defined as follows:

$$r_t = \ln P_t - \ln P_{t-1} = p_t - p_{t-1} \ , \tag{11}$$

$$R_t = \frac{\ln P_t - \ln P_{t-1}}{\mathrm{SD}(r_t)} = \frac{r_t}{\mathrm{SD}(r_t)} = \frac{p_t - p_{t-1}}{\mathrm{SD}(r_t)} \ , \tag{12}$$

where SD(rt) is the standard deviation of the return rt. Notice that the SD(rt) of a given time series rt is a constant variable (assumed equal to C), then, in an efficient market, the normalized return Rt subjects to white noise sequence with zero mean and σ2/C2 variance.

3.2. MApEn test

As the normalized return of the exchange rates subjects to the white noise process in the efficient FX markets, we first compare the MApEn test between on white noise and 1/f noise. The 1/f noise is a popular stochastic process in nature, its spectral density S(f) and frequency f having the form: $S(f)\propto 1/f$. Here we test the simulated white noise and 1/f noise with 5,000 observations, scale factor τ=20, embedding dimension m=2 and similar tolerance r=0.25SD. The MApEn results are shown in Fig. 1. We can find that for scale one, the ApEn value of the white noise series is higher than the 1/f noise time series. However, the ApEn value for the coarse-grained white noise series gradually decreases, while the ApEn value for the coarse-grained 1/f noise time series trends constant level for all scales. This result confirms the fact that, unlike white noise, 1/f noise has complex structures across multiple time scales [17].

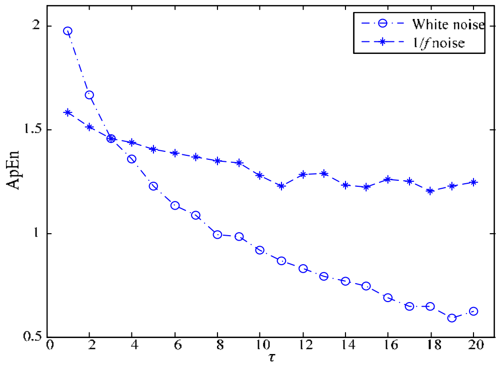

Fig. 1. MApEn analysis of white noise and 1/f noise time series

Based on the above EMH discussion and MApEn test, we can get the conclusion that if the MApEn curve of a specific FX rate is closer to the one of white noise process, the FX market of which is more efficient. On the contrary, if the MApEn curve deviates from white noise curve (e.g. closes to 1/f noise curve), its FX market is inefficient, and remains to be improved.

3.3. Data Source

In order to compare our results to Oh, et al [3], we also use the daily foreign exchange rates for 17 currencies to US Dollar (USD) separately come from Europe, North American, African, Asian, and Pacific Ocean countries. We split the daily FX rates data into three periods by the Asian currency crisis and US sub-prime crisis, the first period from 1984 to 1998, the second period from 1999 to 2008 and the third period form 2009 to June, 2011. The daily FX rates data provided by Board of Governors of the Federal Reserve System (http://www.federalreserve.gov/releases/). The 17 currencies in our study and the respective currency symbols are presented in Table 1. The normalized return Rt is defined by equation (9), (11) and (12).

3.4. ApEn analysis of market efficiency in foreign exchange markets

According to Oh, et al [3], the results of the ApEn test on market efficiency in FX markets are shown in Fig. 2. Fig. 2(a) shows the ApEn values for FX rates in Period I before the Southeast Asian currency crisis. And we find that the average ApEn for Europe, North America and Pacific Ocean FX markets is lager than the one for Asia markets. One of the reasons is due to the liquidity or trading volumes in the developed markets are much lager than emerging markets, as argued by Oh, et al [3]. In Period II, the market efficiency of North America, Africa and Pacific Ocean markets are close to the one of Europe markets. Notably, the ApEn values for Asia markets increase sharply after the Southeast Asian currency crisis except for CNY, HKD, and INR, indicating the improvement of market efficiency (see details in Fig. 2(b)). After the American sub-prime crisis, the ApEn values for Europe, North America, Pacific Ocean, and Africa markets keep on the same level, while the ApEn values for the Asian markets of CNY, HKD and INR increase dramatically, which indicates that the Asian FX markets improved significantly after the financial crisis.

As discussed above, though the ApEn can be an indicator of the market efficiency, it ignores the question how efficiency in different FX markets. In other words, it is not convincible that Europe and North America markets obey the EMH even they have the highest ApEn values. Obviously, what we need is beyond what the ApEn method got. Therefore, in the next section, we will introduce the MApEn method to evaluate of market efficiency by characterizing the complexity structure of FX markets.

Table 1. 17 currencies and respective symbols

Continent	Currencies	Symbol	Continent	Currencies	Symbol
Europe	Swiss	CHF	Asia	Chinese Renminbi	CNY
	Danish Krone	DKK		Hongkong Dollar	HKD
	British Pound	GBP		Indian Rupee	INR
	Norwegian Krone	NOK		Japanese Yen	JPY
	Swedish	SEK		South KoreanWon	KRW
North America	Canadian Dollar	CAD		Singapore Dollar	SGD
Africa	South African Rand	ZAR		Taiwan Dollar	TWD
Pacific Ocean	Australian Dollar	AUD		Thai Baht	THB
	New Zealand Dollar	NZD			

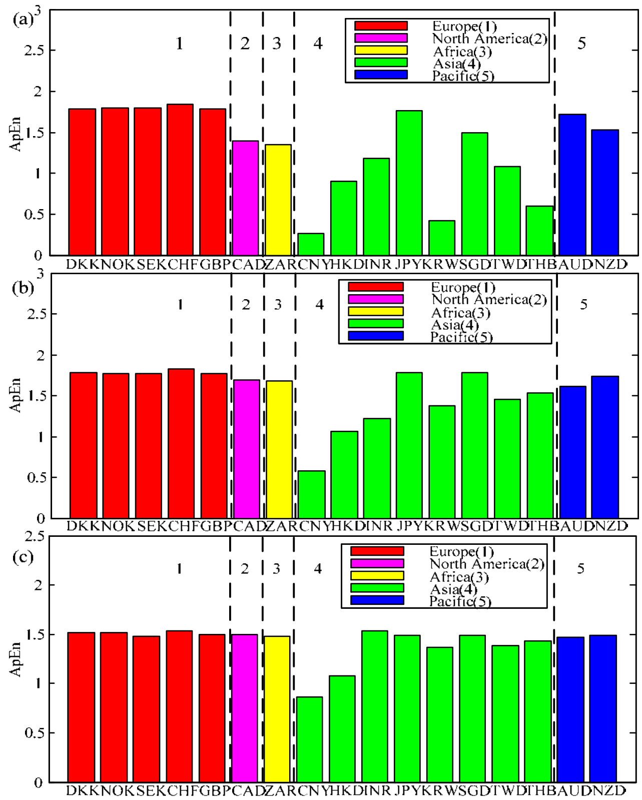

Fig. 2. (a)~(c) ApEn for FX rates of 17 currencies in Period I, II, and III, respectively

3.5. MApEn analysis of market efficiency in foreign exchange markets

In this section, we employ the proposed MApEn approach to analyze market efficiency in the FX markets. The parameters of m, r, τ are as the same as in Section 3.2. And the empirical results are illustrated in Fig. 3.

Before the Southeast Asia currency crisis happens, the MApEn curves are divided into three clusters: (i) the curves for the developed countries of GBP, DKK, AUD and JPY almost superpose to the one for white noise series, which implies those FX markets are high efficient; (ii) The curve of NZD, SGD, ZAR, CAD and INR are close to the white noise curve, which indicates that those market efficiency lower than the former cluster. Notably, we find that the curve of TWD close to the $1/f$ noise trend, which mean its market is significant inefficient. (iii) The curves for the Asian markets of THB, KRW, HKD and CNY almost superpose the $1/f$ noise curve. In sum, the market efficiency of Europe and North American FX markets is higher than Asian (except JPY) and African markets (see details in Fig. 3(a)). The conclusion is consistent with ApEn test.

Fig. 3(b) presents the MApEn test results in Period II, between the occurrences of Southeast Asia currency crisis and American sub-prime crisis. Overall, most of curves, except some Asia currencies of CNY, HKD, INR and KRW, accord with the decreasing trend of white noise series. These phenomena are also consistent with the discussion in ApEn analysis, which suggests that those Asian FX markets (except Japan) need more improvement on market efficiency.

Fig. 3(c) plots the MApEn curves in Period III started with the American sub-prime crisis. It shows that the most of FX markets becomes more efficient. However, the curves of CNY and HKD fluctuate significantly and deviate from the curve of white noise series. And the MApEn curve of KRW fluctuates notably, whereas it locates in the cluster of quasi-white noise. Therefore, the market efficiency of CNY, HKD and KRW is still lower than the one of developed FX markets.

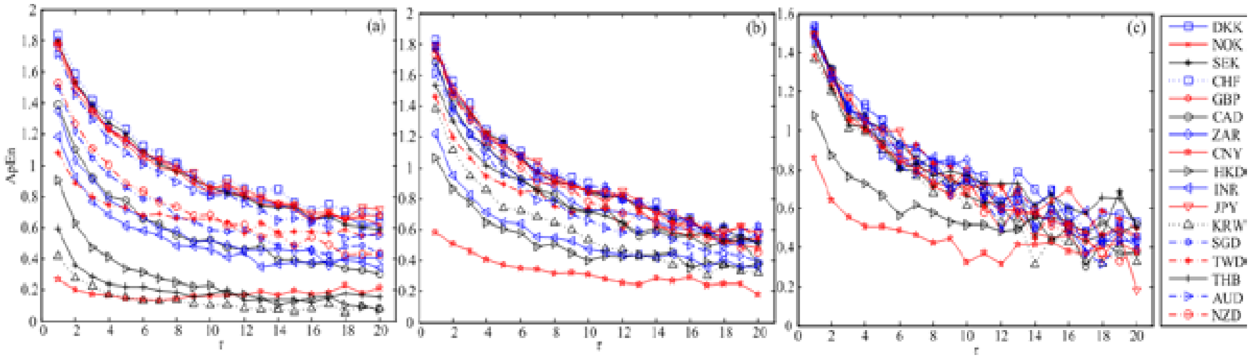

Fig. 3. (a)~(c) MApEn for FX rates of 17 currencies in Period I, II, and III, respectively

Comparing with the ApEn approch, the MApEn test measures the market efficiency of global FX markets and characterizes the complexity of FX rate series more properly. Moreover, the MApEn method provides more information in higher scales than the ApEn method does.

4. Conclusions

This paper investigates the properties of FX rate, introduces the MApEn approach to characterize the market efficiency in 17 FX markets, and discusses the effect of most recent financial crisis on FX market efficiency. The empirical results show that the market efficiency of Europe and North American FX markets is higher than Asian (except Japan) and African FX markets before Southeast Asia currency crisis. Through the American sub-prime crisis, the Asia and African FX markets improved significantly, but several Asia FX markets are still far away from EMH. Going with the American sub-prime crisis, the most of the FX markets shows higher randomness than the ones in Period I and II.

Just as every coin has two sides, despite the global financial crisis brings economic depression, but improves the market efficiency significantly. Especially, the promoting effect is more evident in emerging market, like China, Hong Kong and Korea FX market.

Furthermore, our findings imply that the MApEn approach is a valid measure of the market efficiency. Comparing with the ApEn method, the MApEn test characterizes the foreign exchange market efficiency in higher scale and is more proper to describe market efficiency in FX markets.

5. Copyright

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