



Production process changes: A dynamic programming approach to manage effective capacity and experience

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Received 7 May 2004; accepted 20 August 2004

Abstract

The introduction of process changes is often used by management to invest in production competencies. However, implementing process changes causes disturbances in production learning. This work describes a process change strategy to increase the effective capacity of a production system when unit costs are subject to a learning curve. It is found that the optimal process change level is decreasing in the effective capacity level and increasing in the accumulated knowledge level and production learning rate. Conditions are provided under which the optimal process change level is larger/smaller than the myopic process change level.

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Keywords: Process changes; Learning; Knowledge management; Dynamic programming

1. Introduction

1.1. Problem environment

Manufacturing departments of electronics firms deal with complex and knowledge-intensive production processes. In these environments frequent introductions of changes to the process recipe can be observed. Examples of incremental process changes are equipment changes, implementation of software to support manufacturing, procedural changes, etc. Empirical research shows that such

process change implementations are responsible for important jumps backward on the learning curve. Adler and Clark (1991) show that process changes caused by changes of the product have a disruptive effect on learning through sustained production activities. Marcie and Hauptman (1992) worked on the idea that the introduction of important process changes such as a new technology, is a source of uncertainty and as such disturbances. The two problematic attributes identified from the implementation and usage of new technology: the technical complexity and the shift in production approaches and organizing principles involved in using the new technology. Hatch and Mowery (1998) find that the disruptive

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effects of the introduction of process innovations on learning for the existing process in the semiconductor industry are significant. Lapre et al. (2000) report that process changes due to quality improvement projects without preparation of the work force, disturb the process of waste reduction in production. Terwiesch and Xu (2004) argue that if process specifications are changed and introduced in the production environment, line workers have to adjust to the new situation: behavioural patterns have to be adjusted and new operating procedures have to be developed to cope with the new environment. As indicated by these authors, the implementation of a process change makes some of the accumulated production knowledge such as operating procedures obsolete. New operating procedures have to be developed to handle the modified process recipe. The more significant a process change is, the larger the decrease of the accumulated production knowledge.

1.2. Decision problem

Although empirical evidence shows the negative effect of process changes on learning in production, managements often uses the introduction of process changes to invest in production competencies to boost profits from operating a production system, but face a dilemma: if investment in process change is too low, performance improvement opportunities due to scientific or technological progress will be forgone; on the contrary, a too large investment in process change is responsible for a serious disruption of the production department, which causes losses difficult to earn back.

1.3. Literature review

In the normative operations management literature, Carrillo and Gaimon (2000) present an optimal control model that uses the process change level and the level of training and preparation for process change as variables under control of management. The production environment is parameterised with the level of effective capacity of the production system and the level of knowledge present in the production

environment. In the dynamics of the model, they consider a short-term loss and a long-term gain in effective capacity of a production system due to process change activities. Further, the level of knowledge present in the production environment increases with the level of process change. The level of knowledge increases the performance of the production system. The authors describe the optimal process change policy and the optimal preparation and training policy. The optimal amount of process change over the production horizon is larger if the amount of cumulative knowledge is larger. The optimal preparation and training policy is decreasing in time.

Terwiesch and Xu (2004) include explicitly the loss of knowledge due to process change in an optimal control model. They use production rate, process change rate and learning effort as decision variables to optimise the profit from cumulative process changes and the stock of knowledge. In the law of motion of the stock of knowledge, the level of process change has a negative impact. The optimal learning policy is monotone decreasing in the knowledge level and the optimal process change level is larger for larger amounts of cumulative knowledge.

Comparing the assumptions on the dynamics of both models leads to an interesting result: in the Carrillo model, the level of knowledge increases with the level of process change whereas in the Terwiesch model, the level of knowledge decreases with the level of process change. Carrillo explains the increase through a learning-by-doing effect: while implementing the new process recipe, operators and engineers learn how to do it.

Terwiesch explains his assumption differently: due to the change of the process recipe, a part of the accumulated production experience becomes obsolete which clarifies the empirically established jump backward on the learning curve. One way to interpret the difference between both models is that Carrillo does not consider knowledge generated through sustained production but focuses on knowledge generated through training, preparation for implementation of process changes. The phenomenon described in Terwiesch occurs for knowledge generated through sustained

production and is additive to the phenomena described in Carrillo.

1.4. Contribution

The objective of this paper is to gain further insight in the control of process change activities. Normative research that includes the empirically established knowledge consumption effect of process change is extended and further exploration of optimal process change planning for a manufacturing unit, isolated from competitive environment, and producing a single product is provided. Conditions affecting the planning are: unit production costs decrease with the level of knowledge generated through sustained production while process change consumes knowledge. A general unit cost and process change cost function is considered. Due to the dynamic nature of learning processes, a dynamic model is necessary to analyse optimal process change planning. To prepare for the inclusion of risk in future work, a discrete time horizon is selected opposed to the continuous time horizon used in literature. The decision problem is structured using dynamic programming.

It is found that a myopic producer invests more in process change if the level of production knowledge increases and invests less in process change if the effective capacity level increases. The optimal process change level is increasing in production learning rate and knowledge level and decreasing in effective capacity level. Under the assumptions of the model, necessary conditions are provided under which an optimal policy selects, every period a process change level smaller or larger than the myopic level.

2. Problem structure

In this section the decision problem is structured in a formal way as a preparation for the analysis. The problem structured with a single decision variable, two state variables, a system equation for each state variable, a period reward function and the discounted profit maximisation optimality criterion. The section ends with the statement of the optimisation problem.

2.1. Decision variable

At the beginning of every production period, management sets the process change level p ($p \in P = [0, \bar{p}]$). An upper bound on the level of process change is included to represent a budget constraint. A process change level larger than zero ($p > 0$) results in the implementation of a process change (a change of the process recipe).

2.2. State variable

The problem environment is parameterised with the level of accumulated production knowledge and the level of effective capacity. In the Carrillo model similar state variables are used. From literature it is clear that there is a strong relation between knowledge generated through sustained production and process change activities. Further the positive influence of that knowledge on the production unit cost is also widely described. Therefore production knowledge is selected as a parameter to describe properties of the production environment relevant for the decision-maker. The accumulated knowledge in the production environment s ($s \in S = [0, \infty]$) includes routines developed to implement the current process recipe and perform the resulting production activities.

Under the condition that demand exceeds capacity, which frequently occurs during ramp-up, the managerial objective of changing the process recipe, is increasing the effective capacity level of the current production system. Therefore, the effective capacity level k is selected as a parameter to describe the state of the production system. We assume that k takes values in $K = [0, \infty]$. Effective capacity is also selected as a state variable in Carrillo and Gaimon (2000) and Spence and Porteus (1987).

2.3. System equations

The system equations describe the effect of the decision variable on the state variables.

Two phenomena are included in the knowledge dynamics: production learning and a change of the process recipe. Firstly, the effect of production learning on the accumulated knowledge level is

explained. Due to the repetitive execution of production activities, the firm gains experience with the current production process and accumulates knowledge during a production period. It is assumed that an increase in knowledge is a constant proportion ($\gamma \in]0,1[$) of the present knowledge level s and independent of the capacity level. That assumption is made for analytical purposes and is a linear approximation of a concave phenomenon. The increase in knowledge due to production learning is observed with a one-period time lag as presented in Fig. 1. In the next period, the firm can take advantage of the increase in knowledge through a reduced unit cost. The effect of knowledge depreciation is not included. In the present modelling approach, this effect is opposite to production learning. To include knowledge depreciation, the depreciation rate should be subtracted from the production learning rate γ .

Secondly, the effect of the process change level on the accumulated knowledge level is described. In manufacturing firms frequent changes of the process recipe can be observed. Due to changes in the process recipe, some of the procedures developed to perform activities prescribed in the process recipe become obsolete and have to be replaced by new ones. A decrease of the accumulated knowledge is observed. That phenomenon is also described in Terwiesch and Xu (2004).

The loss of relevant knowledge is proportional ($\beta \in [0,1]$) to the size of the process change. A similar assumption is made in Terwiesch and Xu (2004). β can be interpreted as a coefficient that

translates effect of the level of process change on the knowledge level.

If we assume that a process change is implemented directly after the decision is taken, effect on the knowledge level is instantaneous. If at decision epoch t management chooses a process change level p , a decrease in the knowledge level is observed. A production run starts just after the implementation of the process change. At decision epoch $t+1$, the increase in knowledge due to production experience occurs as shown in Fig. 2.

To ease notational burdens in the following paragraphs, we use $g(s,p) = s(1 - \beta p)$. Due to this formulation and assumed non-negativity of s , the upper bound of p, \bar{p} , must be equal or smaller than $1/\beta$.

The positive effect of the process change level on the performance of the production process on the other hand is measured through the effective capacity. The effect of the process change level on the effective capacity level k is instantaneous as shown in Fig. 3.

2.4. Period reward

The unit production cost is a function of the knowledge level as measured just after the implementation of a process change, $C_1(g(s,p))$ with $C_1(\cdot)$ a decreasing strict convex twice differentiable function and with $C_1 = (0)\bar{c}$ and $\lim_{s \uparrow \infty} C_1(s) = \underline{c} \geq 0$. An example of such a cost function is the classical exponential $C(x) = c_a + c_b \exp(-\delta x)$, with $c_a, c_b \in \mathcal{R}_0^+$, $\delta \in]0, 1[$, where c_a is

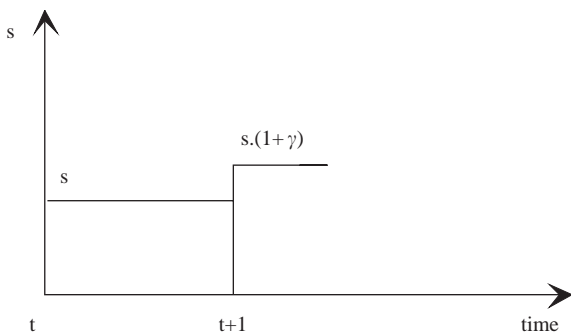


Fig. 1. Effect of learning.

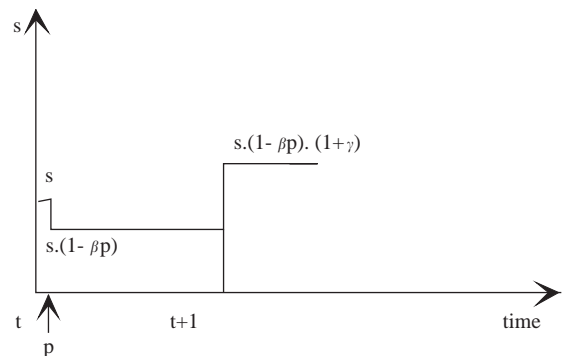


Fig. 2. Combined effect of learning and process change.

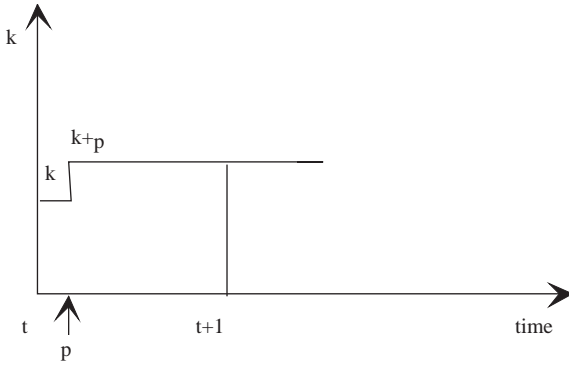


Fig. 3. Effect of process change.

the fixed cost component and c_b is the variable cost component. δ can be interpreted as the rate at which knowledge contributes to a decrease of the variable unit cost. The cost per unit of process change is $C_2(p)$ with $C_2(\cdot)$ an increasing strict convex twice-differentiable function on P such as a quadratic cost function. Because no competitive effects are included in the model, the revenue is kept at a constant level R per capacity unit. It is assumed the decision-maker can make a profit from a process change strategy. If not, the decision-maker will choose $p = 0$ and it would be meaningless to pursue any further analysis. Therefore we assume $R \geq C_1(0)$. The period reward is defined as

$$r(k_{t-1}, s_{t-1}, p_t) = [R - C_1(g(s_{t-1}, p_t))](k_{t-1} + p_t) - C_2(p_t)$$

2.5. Optimisation problem

For $i = 1, 2, \dots$ let $p_i(k, s) : (K \times S) \rightarrow P$ and let $\pi = \{p_1(k, s), p_2(k, s), p_3(k, s), \dots\}$ be a policy with associated reward

$$W^\pi(k, s) = \sum_{t=1}^{\infty} [r(k_{t-1}, s_{t-1}, p_t)].$$

Management's problem is to determine a policy to maximise the associated total discounted reward with $\alpha \in]0, 1[$ the discount factor. The objective function is defined as

$$F(k, s) = \sup_{\pi} W^\pi(k, s)$$

subject to

$$s_t = g(s_{t-1}, p_t)(1 + \gamma),$$

$$k_t = k_{t-1} + p_t,$$

$$k_t, s_t \geq 0, \quad \forall t \in \{0, 1, \dots\}.$$

A policy π^* is an optimal policy if the pay off it generates from any initial state is the supremum over possible pay offs from that state. We assume an infinite planning horizon which is a reasonable assumption if no information is available on the planning horizon, the planning horizon is very long or when successors of the product recipe make use of identical production technologies. The recursive formulation of the problem is

$$V(k, s) = \sup_{p \in P} \{r(k, s, p) + \alpha V[k + p, g(s, p)(1 + \gamma)]\},$$

where $F(k, s)$ is the optimal value function. We show the existence, uniqueness and continuity of the optimal value function, using a contraction mapping argument. Further, we establish the existence and uniqueness of an optimal stationary policy and provide a partial characterisation.

3. Model analysis¹

3.1. Properties of the period reward

The period reward function is continuous, bounded and differentiable on $(K \times S \times P)$. Using calculus, it is straightforward to show that the period reward function is strictly increasing on K and S . Taking advantage of the differentiability assumptions on C_1 and C_2 , it is easy to show, that the period reward function is strictly concave on P . Therefore, the myopic planner is faced with an optimisation problem over a convex constraint set with a strictly concave objective function. As such a unique myopic policy exists. Using the condition for submodularity for differentiable functions, it can be shown that the period reward function is submodular on $(P \times K)$. This can be interpreted as: if the effective capacity level increases, the

¹Proofs of the propositions are omitted and can be obtained from the corresponding author.

decision-maker will decrease his investment in process change. If

$$C_1''(g(s,p))(k+p)s\beta(1-\beta p) - C_1'(g(s,p))[(1-\beta p) - \beta(k+p)] \geq 0$$

holds for every s and p , the period reward is supermodular on $(P \times S)$, that is if the knowledge level increases, a myopic planner will increase his investment in process change. If $C_1(\cdot)$ is replaced with an exponentially decreasing unit cost function $C_1(\cdot) = c_b \exp(-\delta)$, that condition can be simplified to

$$s \geq \frac{(k+p)\beta - (1-\beta p)}{\delta(k+p)\beta(1-\beta p)}.$$

Analysis of the condition for relevant values of k , p , δ and β shows that it is fulfilled even for s close to zero. For a process change level approaching \bar{p} , the condition is not fulfilled because

$$\lim_{p \uparrow 1/\beta} \frac{(k+p)\beta - (1-\beta p)}{\delta(k+p)\beta(1-\beta p)} = \infty^+.$$

For further analysis, one assumption is needed: the jointly strict concavity on $(K \times S \times P)$ of the period reward function.

3.2. Properties of the optimal value function

The behaviour of the optimal value function is described using real analysis and a contraction mapping argument. That approach is well-described in the textbooks of [Stokey and Lucas \(1989\)](#), [Sundaram \(1996\)](#) and [Smith and McCardle \(2002\)](#). A good illustration of the approach can be found in [Mazzola and McCardle \(1997\)](#). For the proofs of the derived properties in this paper similar arguments are used as in the above references and therefore only the key arguments of the proofs are given. In this section the existence and properties of the optimal value function such as continuity, strictly increasing and strictly concave are established.

Because the period reward function is bounded and the discount factor is less than one, the optimal value function is well-defined. Using a contraction mapping argument, the value function is the unique bounded solution of the Bellman equation and is continuous. Using again the

contraction mapping argument, the fact that the period reward function is strictly increasing on $(K \times S)$ and that the set of bounded continuous strictly increasing functions is closed, it is easy to show that the optimal value function is strictly increasing on the state space. Using the assumption of strict concavity of the period reward function, a similar approach can be used to show the strict concavity of the value function on the state space.

3.3. Existence and characterisation of an optimal policy

In this paragraph the existence of a unique optimal policy is established and properties of that policy are described. The information that the planning horizon is infinite, limits the set of feasible policies. An optimal policy will be stationary and of the form $\pi = \{p(k,s), p(k,s), p(k,s), \dots\}$. As derived in the previous section, the optimal value function $F(k,s)$ exists and is a solution of the Bellman equation. The Weierstrass theorem shows the supremum is attained over the domain, therefore an optimal policy exists that generates the optimal value function.

To describe the optimal policy, we make use of the theory of correspondences. A good introduction to the material can be found in [Sundaram \(1996\)](#). The action correspondence gives the set of feasible actions of the decision-maker and is defined by Φ . Because $\Phi(k,s) = P$ for every (k,s) , Φ is a constant valued correspondence, further P is bounded and compact valued. As such the feasible action correspondence is continuous. We define the graph of Φ by

$$\text{Gr}(\Phi) = \{(k,s,p) \in K \times S \times P | p \in \Phi(k,s)\}.$$

A constant valued action correspondence has a convex graph. We define the optimal action correspondence as

$$G(k,s) = \{p \in P | F^*(k,s) = r(k,s,p) + \alpha F^*(k+p, g(s,p)(1+\gamma))\}.$$

Because an optimal policy exists, G is non-empty. With the strict concavity of the optimal value function, all the conditions of the maximum

theorem under convexity are fulfilled and we can conclude that $G(k, s)$ is single valued everywhere and therefore a continuous function. In the following paragraph the unique optimal policy is further described.

To describe the behaviour of the optimal policy in the effective capacity level, knowledge level and production learning rate, we make use of the approach described by Santos (1991). The Euler condition² and the implicit function theorem are fundamental in that approach. With the existence of a unique optimal policy function and if every period the process change level is interior, the optimal policy $\{p_t\}$ satisfies the Euler necessary condition for an optimum,

$$\frac{\delta}{\delta y} [r(k_{t-1}, s_{t-1}, y) + \alpha r(k_{t-1} + y, s_{t-1}(1 - \beta y) \\ (1 - \gamma), p_{t+1})] = 0.$$

In the Euler condition, the optimal policy function is implicitly defined. Because the conditions of the implicit function theorem are fulfilled, we can use the theorem to determine the sign of the partial derivative of the optimal policy function with respect to k , s and γ . After some algebra, we find that the sign of the partial derivative of the optimal policy function with respect to k is negative. Knowing that the myopic policy is submodular on $(K \times P)$, the result is not surprising and the optimal process change level decreases in the level of effective capacity. The behaviour of the optimal policy function in the knowledge level also echoes the behaviour of the myopic policy. For the exponentially decreasing unit cost function $C_1(.) = c_b \exp(-\delta.)$, the conditional behaviour can be characterised as: if the myopic policy is super-modular and if

$$s > \frac{\beta(k + p + p_{t+1}) - (1 - \beta p)}{\delta \beta(k + p + p_{t+1})(1 - \beta p)(1 - \beta p_{t+1})(1 + \gamma)},$$

the optimal policy is increasing in the knowledge level. Analysis of the condition for relevant levels of k, s, p, δ, β and p_{t+1} shows that it holds even for very small s , though for a process change level

approaching \bar{p} , the condition is not fulfilled because

$$\lim_{p \uparrow 1/\beta} \frac{\beta(k + p + p_{t+1}) - (1 - \beta p)}{\delta \beta(k + p + p_{t+1})(1 - \beta p)(1 - \beta p_{t+1})(1 + \gamma)} \\ = \infty^+.$$

The behaviour of the optimal policy function in the production learning rate is described by the following condition. If

$$\gamma > \frac{1}{\delta s(1 - \beta p)(1 - \beta p_{t+1})} - 1$$

holds for γ in $]0, 1]$, the optimal policy function is increasing in production learning rate. Analysis of the condition for relevant levels of p, s, δ, β and p_{t+1} shows that it holds, though for p approaching \bar{p} , the condition obviously fails.

The relation with the myopic policy gives information on the structure of the optimal policy. With \hat{p} the myopic policy and p^* the optimal policy and using optimality we know that

$$r(k, s, p^*) + \alpha F^*[k + p^*, g(s, p^*)(1 + \gamma)] \\ \geq r(k, s, \hat{p}) + \alpha F^*[k + \hat{p}, g(s, \hat{p})(1 + \gamma)].$$

Because $r(k, s, \hat{p}) \geq r(k, s, p^*)$, then surely

$$F^*[k + p^*, g(s, p^*)(1 + \gamma)] \\ \geq F^*[k + \hat{p}, g(s, \hat{p})(1 + \gamma)]$$

must hold. $F(., .)$ is now strictly increasing, thus if $k + p^* \geq k + \hat{p}$ and $g(s, p^*) \geq g(s, \hat{p})$ and thus $p^* \geq \hat{p}$ and $\hat{p} \geq p^*$, the above weak inequality surely holds. Therefore we can conclude that if $p^* = \hat{p}$, the necessary condition of an optimum holds. Because the argument is based on a necessary condition for an optimum, we have to check for $p^* \neq \hat{p}$.

Case 1: $p^* > \hat{p}$.

A necessary condition for $p^* > \hat{p}$ to hold is the increase of F^* in the first argument due to the selection of p^* over \hat{p} which is larger than the decrease of F^* in the second argument due to the selection of p^* over \hat{p} . Under interiority restrictions on the state variables and decision variable, $F^*(., .)$ is continuously differentiable and the above

²This Euler condition is a discrete time version of the classic Euler condition used in the calculus of variations.

statement can be reformulated as

$$\left[\frac{\partial}{\partial x} F^*(x, y^*) \right]_{x=x^*} - \left[\frac{\partial}{\partial x} F^*(x, \hat{y}) \right]_{x=\hat{x}} > \left[\frac{\partial}{\partial y} F^*(x^*, y) \right]_{y=y^*} - \left[\frac{\partial}{\partial y} F^*(\hat{x}, y) \right]_{y=\hat{y}}$$

with

$$\hat{x} = k + \hat{p}, \hat{y} = g(s, \hat{p})(1 + \gamma), \\ x^* = k + p^*, y^* = g(s, p^*)(1 + \gamma).$$

Expressed in the period rewards, we have

$$\left[\frac{\partial}{\partial x} r(x, y^*, p(x, y^*)) \right]_{x=x^*} - \left[\frac{\partial}{\partial x} r(x, \hat{y}, p(x, \hat{y})) \right]_{x=\hat{x}} > \left[\frac{\partial}{\partial y} r(x^*, y, p(x^*, y)) \right]_{y=y^*} - \left[\frac{\partial}{\partial y} r(\hat{x}, y, p(\hat{x}, y)) \right]_{y=\hat{y}}.$$

Case 2: $p^* < \hat{p}$.

A necessary condition for $p^* < \hat{p}$ to hold is the decrease of F^* in the first argument due to the selection of p^* over \hat{p} which is smaller than the increase of F^* in the second argument due to the selection of p^* over \hat{p} , that is

$$\left[\frac{\partial}{\partial y} F^*(\hat{x}, y) \right]_{y=\hat{y}} - \left[\frac{\partial}{\partial y} F^*(x^*, y) \right]_{y=y^*} > \left[\frac{\partial}{\partial x} F^*(x, y^*) \right]_{x=x^*} - \left[\frac{\partial}{\partial x} F^*(x, \hat{y}) \right]_{x=\hat{x}}.$$

To illustrate these conditions, consider a finite planning horizon and a large salvage value for capacity versus a small salvage value for knowledge. Under this condition, an optimal policy invests more in process change than a myopic one. Under the opposite case an optimal policy invests less in process change than a myopic one.

Finally, we give an insight into the behaviour of the state sequences $\{k_i\}$ and $\{s_i\}$ generated by the optimal policy. From the law of motion of the effective capacity level k , it is clear that $\{k_i\}$ is increasing. Convergence of the sequence is only possible if p^* drops to zero.

Before describing the behaviour of $\{s_i\}$, it is important to notice that if the system reaches the set of states $(k, s = 0)$, it cannot escape it and the knowledge level provides no information to the decision-maker. Only the effective capacity level can cause a difference between the myopic and

optimal process change level. To assure the set $(k, s = 0)$ is never reached, the following condition should hold:

$$s(1 - \beta p^*)(1 + \gamma) > s \text{ or} \\ p^*(k, s) < \gamma / [\beta(1 + \gamma)] \text{ for } s \text{ close to zero.}$$

The process change level works on the knowledge level as well as the effective capacity level: profit can increase through unit cost reduction as well as through an increase in effective capacity. The information that knowledge will increase through production experience and will decrease through implementation of process change affects the optimal policy. If the condition does not hold, the knowledge level will fall to zero. Then profit can only increase through an increase of the effective capacity level and knowledge dynamics do not influence the optimal policy. Convergence of the sequence $\{s_i\}$ occurs if $p^* = \gamma / [\beta(1 + \gamma)]$.

3.4. Comparative statics

For the exponentially decreasing unit cost function $C_1(s, p) = c_a + c_b \exp(-\delta s(1 - \beta p))$, the value function is increasing with R and δ and decreasing with c_a, c_b and β . The myopic policy is also increasing with δ . For the numerical example with $R = 1000$, $c_a = 0$, $c_b = 50$, $k = 1$, $s = 10$, $\beta = 0.8$, $C_2 = 100$, δ increasing from 0.4 to 0.8, the behaviour of the myopic policy can be seen from Fig. 4. The left-hand graph shows the period reward function. The lower curve, $j(p)$, is the period reward for δ equal to 0.4. In the right hand graph, the derivatives are shown. Again the lower curve, $d/dp j(p)$, has $\delta = 0.4$.

4. Conclusions and further research

In this paper a multi-period decision model is introduced for a decision-maker that parameterises the environment with the level of production experience and the level of effective capacity of the production system. By choosing the process change level every period, the decision-maker maximises the profit of operating the production system. With an infinite planning horizon and for

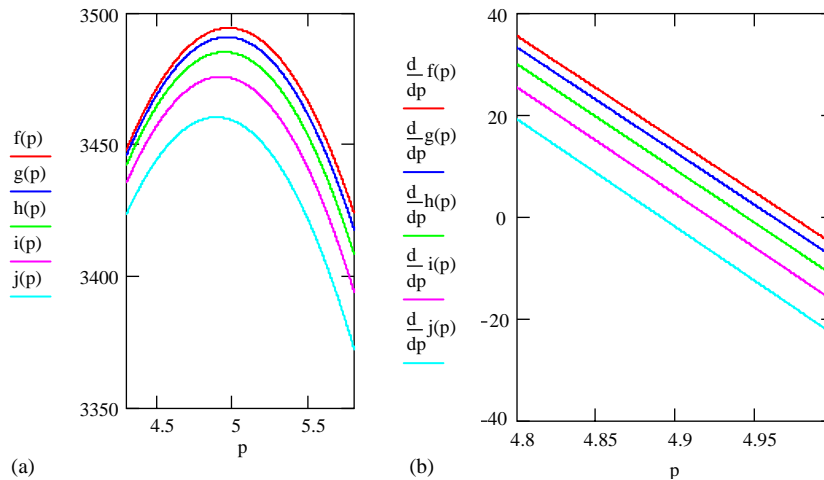


Fig. 4. (a) Combined effect of delta and the level of process change on the period reward; (b) Effect of delta on the derivative of the period reward with respect to the level of process change.

process changes with instantaneous effects on the knowledge level and effective capacity level, an optimal policy will not always invest more in process change than a myopic policy. For a finite planning horizon, dependent on the difference in salvage value of capacity versus knowledge, an optimal policy will invest more or less in process change than a myopic policy. The myopic process change level is increasing in the knowledge level and decreasing in the effective capacity level. The optimal process change level is increasing in the knowledge level and production learning rate and decreasing in the effective capacity level. Further research will try to describe the structure of the optimal policy more precise through analysis of stability. Also the effects of the initial level of effective capacity and knowledge on the state sequences are interesting to analyse. A natural extension is the inclusion of random shocks or risk on the effect of the process change level on the knowledge level or effective capacity level. Another extension is observation noise on the knowledge level.

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